CHAPTER 12

	Big Idea of the Chapter: Find Surface Area and Volume	
	Learning Target	**Success Criteria**
Chapter 12 Surface Area and Volume	Understand surface area and volume.	• Classify solids. • Find surface areas and volumes of solids. • Find missing dimensions of solids. • Solve real-life problems involving surface area and volume.
12.1 Cross Sections of Solids	Describe and draw cross sections.	• Classify solids. • Describe and draw cross sections. • Solve real-life problems involving cross sections.
12.2 Volumes of Prisms and Cylinders	Find and use volumes of prisms and cylinders.	• Find volumes of prisms and cylinders. • Solve real-life problems involving volumes of prisms and cylinders. • Find surface areas and volumes of similar solids.
12.3 Volumes of Pyramids	Find and use volumes of pyramids.	• Find volumes of pyramids. • Use volumes of pyramids to find measures. • Find volumes of similar pyramids. • Find volumes of composite solids containing pyramids.
12.4 Surface Areas and Volumes of Cones	Find and use surface areas and volumes of cones.	• Find surface areas of cones. • Find volumes of cones. • Find the volumes of similar cones. • Find the volumes of composite solids containing cones.
12.5 Surface Areas and Volumes of Spheres	Find and use surface areas and volumes of spheres.	• Find surface areas of spheres. • Find volumes of spheres. • Find the volumes of composite solids.
12.6 Modeling with Surface Area and Volume	Understand the concept of density and modeling with volume.	• Explain what density means. • Use the formula for density to solve problems. • Use geometric shapes to model objects. • Solve modeling problems.
12.7 Solids of Revolution	Sketch and use solids of revolution.	• Sketch and describe solids of revolution. • Find surface areas and volumes of solids of revolution. • Form solids of revolution in the coordinate plane.

Go online to access chapter standards and suggested pacing.

COHERENCE Through the Grades

Prior Learning	Current Learning	Future Learning
Grade 7 • **Section 5.2:** Find unit rates for rates involving fractions. • **Sections 9.1–9.3:** Use formulas to find the circumference and area of a circle. Solve mathematical and real-life problems involving perimeters and areas of composite figures. • **Sections 10.1 and 10.3–10.5:** Solve mathematical and real-life problems involving surface areas and volumes of prisms and pyramids. • **Section 10.2:** Solve mathematical and real-life problems involving surface areas of cylinders. • **Section 10.6:** Describe cross sections of three-dimensional figures. **Grade 8** • **Sections 10.1–10.3:** Solve mathematical and real-life problems involving volumes of cylinders, cones, and spheres. **Geometry** • **Section 1.4:** Use measures of geometric shapes to describe objects. • **Sections 11.1 and 11.2:** Understand the formulas for the circumference and area of a circle and use them to find measures. • **Section 11.1:** Find arc lengths and use arc lengths to find measures. • **Section 11.2:** Derive the formula for the area of a sector of a circle. Find areas of sectors of circles and solve problems involving areas of sectors. • **Section 11.3:** Find areas of rhombuses, kites, and regular polygons. • **Section 11.4:** Use two-dimensional figures to model and solve real-life problems. • **Section 11.4:** Use area formulas to find population densities and solve density problems.	**Geometry Chapter 12** • **Section 12.1:** Describe and draw cross sections of solids. • **Sections 12.2–12.4:** Form conjectures for the formulas for the volumes of cylinders, pyramids, and cones. • **Sections 12.2–12.5:** Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems. • **Section 12.6:** Use three-dimensional figures to model and solve real-life problems. • **Section 12.6:** Use volume formulas to find densities in modeling situations. • **Section 12.7:** Sketch and describe solids of revolution.	**Future Courses** • Recognize a conic section as the intersection of a plane and a double-napped cone. • Write equations of parabolas, ellipses, and hyperbolas in standard form. • Classify conic sections from their general equations and by using the discriminant.

Mathematics of the Chapter



Paul's Insights

Listen to Paul talk about how this chapter fits in with your students' learning progressions.

Digital Experience

Explore additional resources that provide professional insights from our authors, or help engage students in their learning using the powerful search and browse feature.

Paul's Notes

Overview

What we're doing…

This chapter presents measurements of solids. Some of the surface area and volume formulas may be familiar to students from middle school, but new formulas will be introduced. The first section introduces students to cross sections of solids, which will help students develop the spatial skills needed to visualize relationships between two-dimensional and three-dimensional figures. Students will further improve these skills at the end of the chapter when they learn to sketch and describe solids of revolution.

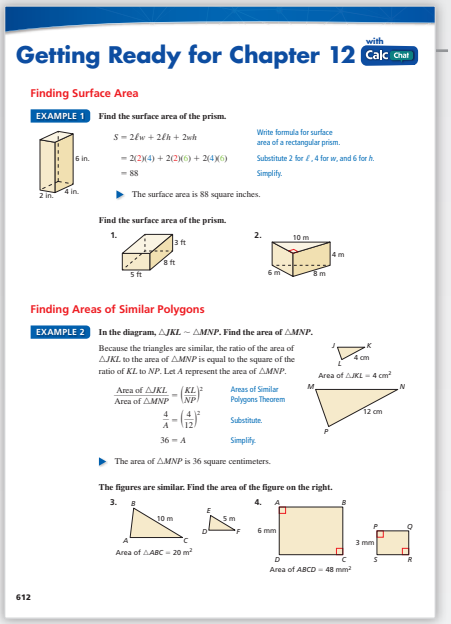
In between these sections, students will undertake an extensive study of volumes and surface areas of prisms, cylinders, pyramids, cones, and spheres. Then students will apply this understanding to modeling with surface area and volume as well as the concept of density.

Why we're doing it…

Students will see that surface area and volume are prevalent among many disciplines and careers, particularly those that work closely with physical models or require spatial reasoning. This includes builders, bakers, sculptors, X-ray technicians, materials engineers, and so many more. For instance, an X-ray technician may need to capture and interpret the cross section of a person's heart.

Essential Background

Students' prior study of surface area and volume will support their understanding of new formulas and concepts presented in this chapter.



Getting Ready for Chapter 12 with CalcChat

Finding Surface Area

EXAMPLE 1 Find the surface area of the prism.

$S = 2\ell w + 2\ell h + 2wh$ Write formula for surface area of a rectangular prism.

$= 2(2)(4) + 2(2)(6) + 2(4)(6)$ Substitute 2 for ℓ, 4 for w, and 6 for h.

$= 88$ Simplify.

▶ The surface area is 88 square inches.

Find the surface area of the prism.

1. 3 ft, 8 ft, 5 ft

2. 10 m, 4 m, 6 m, 8 m

Finding Areas of Similar Polygons

EXAMPLE 2 In the diagram, $\triangle JKL \sim \triangle MNP$. Find the area of $\triangle MNP$.

Because the triangles are similar, the ratio of the area of $\triangle JKL$ to the area of $\triangle MNP$ is equal to the square of the ratio of KL to NP. Let A represent the area of $\triangle MNP$.

$\frac{\text{Area of } \triangle JKL}{\text{Area of } \triangle MNP} = \left(\frac{KL}{NP}\right)^2$ Areas of Similar Polygons Theorem

$\frac{4}{A} = \left(\frac{4}{12}\right)^2$ Substitute.

$36 = A$ Simplify.

▶ The area of $\triangle MNP$ is 36 square centimeters.

Area of $\triangle JKL = 4$ cm^2; 4 cm; 12 cm

The figures are similar. Find the area of the figure on the right.

3. 10 m, 5 m; Area of $\triangle ABC = 20$ m^2

4. 6 mm, 3 mm; Area of $ABCD = 48$ mm^2

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Student Edition page 612

Getting Ready

When finding the surface area of a prism, some students may benefit from drawing a net of the prism. Remind students that the ratio of the perimeters of two similar polygons is equal to the ratio of their corresponding side lengths, but the ratio of the areas of two similar polygons is equal to the square of the ratio of their corresponding side lengths.

- Finding Surface Area
  - Use a formula to find the surface area of a prism. **Grade 7 Section 10.1**
- Finding Areas of Similar Polygons
  - Find areas of similar polygons. **Geometry Section 8.1**

Differentiated Resources

Access suggested resources to reinforce or reteach prior skills needed for this chapter. The resources offered online provide a wide range of layered support for the diverse needs in your classroom.

RESOURCES



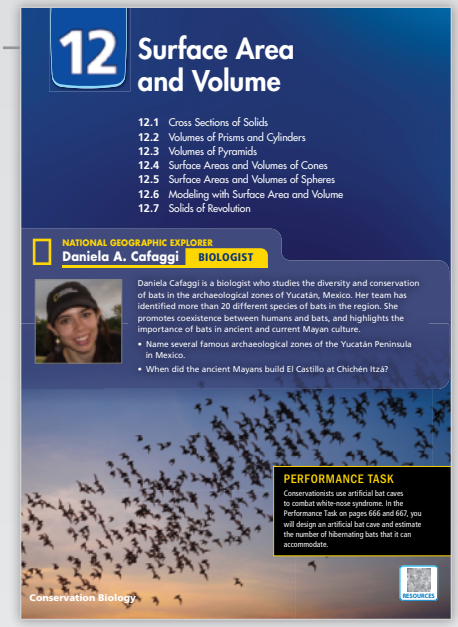
India's Notes

Talk About Careers

"Conservation biologists work to protect endangered plants and animals with efforts focusing on conserving natural environments and wildlife habitats. Conservation biologists may work for state or federal governments, animal parks, private corporations, or non-profit organizations. Many conservation biologists have an area of specialty such as a marine resources specialist or an invasive weed mapping technician."

? "What kinds of conservation efforts might a marine resources specialist focus on?"

? "Why is it important to know where invasive plants grow?"



Student Edition page 610

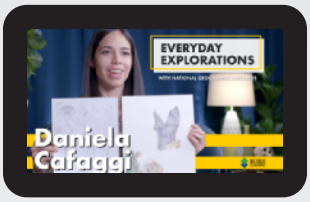
Launch the Chapter with the Career

Discuss the prompt and question on Student Edition page 610. This discussion should spark students' interest and promote thinking about conservation biology.

- *Sample answer:* Some famous archaeological zones in the Yucatán Peninsula are Chichén Itzá, Uxmal, Mayapan, and Cobá.
- The ancient Mayans built El Castillo at Chichén Itzá between the 8th and 12th centuries.

Conclude the Chapter with the Career

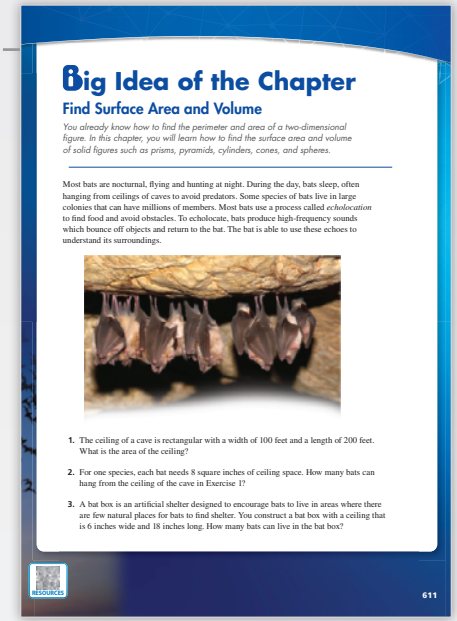
White-Nose Syndrome on Student Edition pages 666 and 667 is a rigorous application that asks students to analyze a data display and apply concepts from the chapter to a real-life situation. Students will use their understanding of surface area, volume, and population density to design a suitable artificial cave for a bat colony.



Hear from a National Geographic Explorer **Biologist**.

Big Idea of the Chapter:
Find Surface Area and Volume

"Populations of animals and people can be packed together or spread out. What do you know about density? What objects could be described as *dense*?" *Sample answer:* Density describes how much matter is packed into an area or volume; *Sample answer:* A popcorn kernel could be described as dense. Have students complete the exercises to explore surface area and how many bats can fit in a given area.



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EL English Learner Support

Interpreting Mathematical Explanations

- Have EL students work with a small group of native speakers to complete this activity.
- Allow the use of translation tools. Encourage students to form their answers in their home language and then translate their answers into English. Assist students with any grammatical errors.

12.1 Cross Sections of Solids

Learning Target:
Describe and draw cross sections.

Success Criteria:
- Classify solids.
- Describe and draw cross sections.
- Solve real-life problems involving cross sections.

Content Standards for Mathematics

Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.

WIDA English Language Development Standards

ELD-MA.9-12.Explain.Expressive Construct mathematical explanations that state reasoning used to generate own or alternate solutions

ELD-MA.9-12.Explain.Interpretive Interpret mathematical explanations by evaluating rationales, models, and/or interpretations based on evidence and mathematical principles

Prerequisite Skills

Classify and describe polygons. **Geometry Section 1.4**

Paul's Notes

Launch

Goal: Students will identify geometric cross sections of real-life objects.

- Bring a banana and a stick of butter to class.

? Hold up the stick of butter so that the longest edges are horizontal. Ask, "If I cut this stick of butter from top to bottom in the middle of the stick, what shape will the exposed surface be? What if I cut the butter diagonally?" a square; a rectangle

? Hold up the banana horizontally and ask, "If I cut this banana from top to bottom in the middle, what shape will the exposed surface be? What if I cut the banana diagonally?" a circle; an oval or ellipse

Talk About It

"In this section, you will learn about these exposed surfaces, which are called *cross sections*."

Use **Quick Check** exercises to quickly identify where students are in their learning.

Paul's Notes

Investigate

Goal: Students will describe cross sections of food.

- Students should have little difficulty with identifying the shape of each cross section in Exercise 1, but listen to how they describe the solid that models each food. For instance, the cucumber is most closely modeled by a composite solid that involves a cylinder and two hemispheres.
- Allow ample time for pairs to compare their descriptions with other pairs. Then engage in a class discussion.
- Explain the meaning of the term *cross section*, which will be formally defined in the lesson as the intersection of a solid and a plane.
- Ask volunteers to share their drawings for Exercise 3 with the class.

? "Where have you seen a cross section used in a real-life situation?" *Sample answer: in a diagram of a cross section of a heart when studying anatomy*

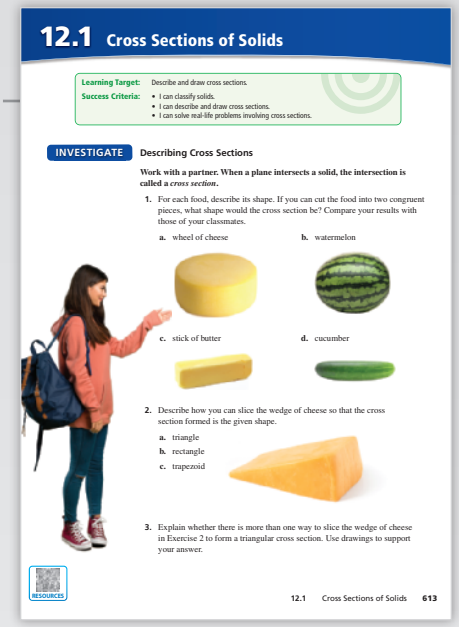
? SMP.7 "When making a cut, what are some things that affect the shape of the cross section?"

Talk About It

"In the lesson, you will review attributes of solids and learn how to describe and draw their cross sections. The pieces of food in the Investigate may have reminded you of particular types of solids."

Differentiating Instruction

- **Emerging:** Students have difficulty visualizing figures in three dimensions, particularly when describing and drawing cross sections. Physical models can help students make sense of the vocabulary and figures in the lesson. Students may also benefit from using dynamic geometry software that allows them to rotate figures in three dimensions.
- **Proficient:** Students have the spatial skills needed to visualize three-dimensional figures and describe cross sections. Drawing cross sections may still be challenging. Have students use dynamic geometry software to rotate figures in three dimensions and gain a better understanding of the possible cross sections.



12.1 Cross Sections of Solids

Learning Target: Describe and draw cross sections.
Success Criteria:
- I can classify solids.
- I can describe and draw cross sections.
- I can solve real-life problems involving cross sections.

INVESTIGATE Describing Cross Sections

Work with a partner. When a plane intersects a solid, the intersection is called a *cross section*.

1. For each food, describe its shape. If you can cut the food into two congruent pieces, what shape would the cross section be? Compare your results with those of your classmates.

a. wheel of cheese b. watermelon

c. stick of butter d. cucumber

2. Describe how you can slice the wedge of cheese so that the cross section formed is the given shape.

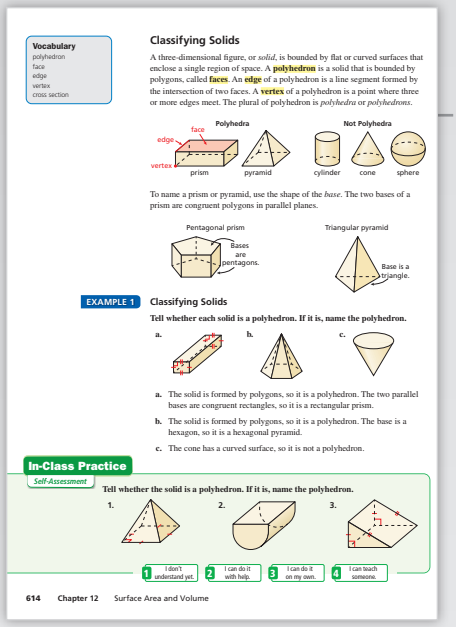
a. triangle
b. rectangle
c. trapezoid

3. Explain whether there is more than one way to slice the wedge of cheese in Exercise 2 to form a triangular cross section. Use drawings to support your answer.

RESOURCES

12.1 Cross Sections of Solids 613

Student Edition page 613



Vocabulary
polyhedron
face
edge
vertex
cross section

Classifying Solids

A three-dimensional figure, or *solid*, is bounded by flat or curved surfaces that enclose a single region of space. A **polyhedron** is a solid that is bounded by polygons, called **faces**. An **edge** of a polyhedron is a line segment formed by the intersection of two faces. A **vertex** of a polyhedron is a point where three or more edges meet. The plural of polyhedron is *polyhedra* or *polyhedrons*.

Polyhedra
face
edge
vertex
prism
pyramid
Not Polyhedra
cylinder
cone
sphere

To name a prism or pyramid, use the shape of the *base*. The two bases of a prism are congruent polygons in parallel planes.

Pentagonal prism
Bases are pentagons.
Triangular pyramid
Base is a triangle.

EXAMPLE 1 Classifying Solids

Tell whether each solid is a polyhedron. If it is, name the polyhedron.

a. The solid is formed by polygons, so it is a polyhedron. The two parallel bases are congruent rectangles, so it is a rectangular prism.

b. The solid is formed by polygons, so it is a polyhedron. The base is a hexagon, so it is a hexagonal pyramid.

c. The cone has a curved surface, so it is not a polyhedron.

In-Class Practice
Self-Assessment

Tell whether the solid is a polyhedron. If it is, name the polyhedron.

1. I don't understand yet. 2. I can do it with help. 3. I can do it on my own. 4. I can teach someone.

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Quick Check Exercise 1

Paul's Notes

Classifying Solids

- Discuss the vocabulary used to classify solids. Show students solids that are polyhedra and solids that are not. Then discuss the differences between the two types of solids. Explain the importance of using the shape of the base to classify a prism or pyramid.
- Pose Example 1 and allow time for students to analyze each solid.
- Physical models can be very helpful. If you have models of the solids in Example 1, pass them around so that students can examine them and gain a better understanding of the associated vocabulary.

EL English Learner Support

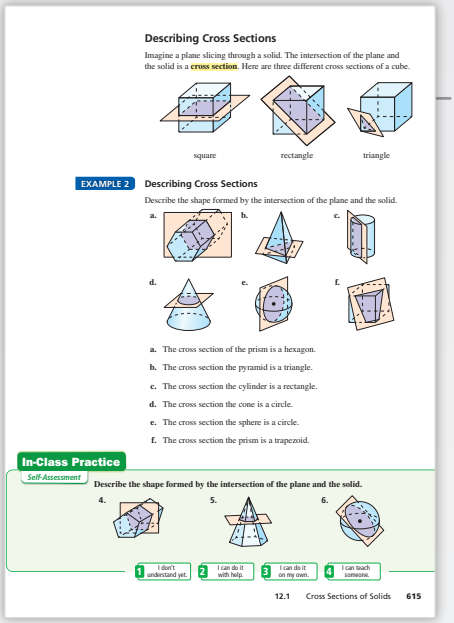
Constructing Mathematical Explanations

As you work through the In-Class Practice exercises, ensure EL students add the vocabulary terms to their math notebooks. Encourage students to include examples and translations in their home language as needed. An *Example and Non-Example Chart* will also be a useful tool for students. As EL students complete Exercises 1–6, provide opportunities for them to verbalize or explain their answers. Support students in the following ways.

Entering–Emerging: Provide sentence starters such as "The solid (is, is not) a polyhedron." and "The solid is called a _____." Students may write the sentences and point to each word as they read them aloud to a partner, repeating several times.

Developing–Expanding: Have partners verbalize their answers and reasoning. Provide sentence starters if needed.

Bridging–Reaching: Have partners share their answers and reasoning using complete sentences.



Describing Cross Sections

Imagine a plane slicing through a solid. The intersection of the plane and the solid is a **cross section**. Here are three different cross sections of a cube.

square
rectangle
triangle

EXAMPLE 2 Describing Cross Sections

Describe the shape formed by the intersection of the plane and the solid.

a. The cross section of the prism is a hexagon.

b. The cross section the pyramid is a triangle.

c. The cross section the cylinder is a rectangle.

d. The cross section the cone is a circle.

e. The cross section the sphere is a circle.

f. The cross section the prism is a trapezoid.

In-Class Practice
Self-Assessment

Describe the shape formed by the intersection of the plane and the solid.

4. 5. 6.

1. I don't understand yet. 2. I can do it with help. 3. I can do it on my own. 4. I can teach someone.

12.1 Cross Sections of Solids 615

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Paul's Notes

Describing Cross Sections

- Students may not realize that a solid can have multiple cross sections. Refer back to the food in the Launch and Investigate to show students some different possibilities.
- Have students work through Example 2. Point out the shading of each cross section to assist students who have trouble visually identifying a cross section.

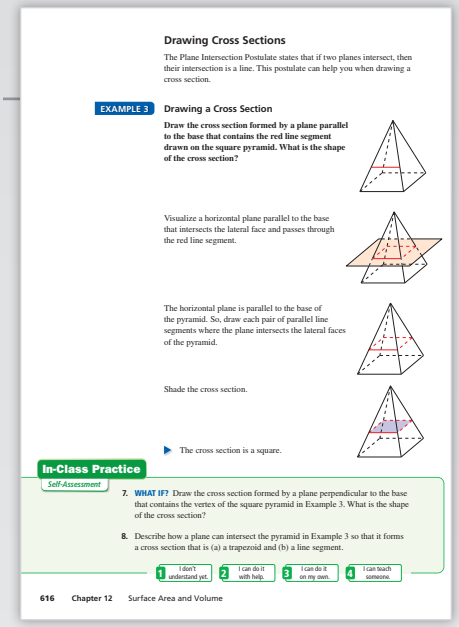
Paul's Notes

Drawing Cross Sections

- Drawing cross sections can be difficult for students. Discuss the Plane Intersection Postulate and guide students through the drawings in Example 3.
- Some students may need to begin by sketching a plane and then wrapping the outline of the cross section around the solid, as shown in Example 3. Other students may feel comfortable with drawing each side of the shape of the cross section as it intersects each lateral face.

Talk About It

"You have learned to classify solids and describe and draw cross sections. Was there a specific example that helped you gain understanding?"



Drawing Cross Sections

The Plane Intersection Postulate states that if two planes intersect, then their intersection is a line. This postulate can help you when drawing a cross section.

EXAMPLE 3 Drawing a Cross Section

Draw the cross section formed by a plane parallel to the base that contains the red line segment drawn on the square pyramid. What is the shape of the cross section?

Visualize a horizontal plane parallel to the base that intersects the lateral face and passes through the red line segment.

The horizontal plane is parallel to the base of the pyramid. So, draw each pair of parallel line segments where the plane intersects the lateral faces of the pyramid.

Shade the cross section.

▶ The cross section is a square.

In-Class Practice

Self-Assessment

7. **WHAT IF?** Draw the cross section formed by a plane perpendicular to the base that contains the vertex of the square pyramid in Example 3. What is the shape of the cross section?

8. Describe how a plane can intersect the pyramid in Example 3 so that it forms a cross section that is (a) a trapezoid and (b) a line segment.

1 I don't understand yet. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone.

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Quick Check Exercise 7

Paul's Notes

Connections to Real Life

- Have students read Example 4 independently. Then have partners discuss a plan for a solution. Listen for each pair to describe using the Pythagorean Theorem to find the width of the cross section.
- Use *Question Box* to assess students' understanding of the lesson and answer any lingering questions.

EL English Learner Support

Interpreting Mathematical Explanations

Have EL students work in small groups to complete Exercise 9. Support EL students in the following ways.

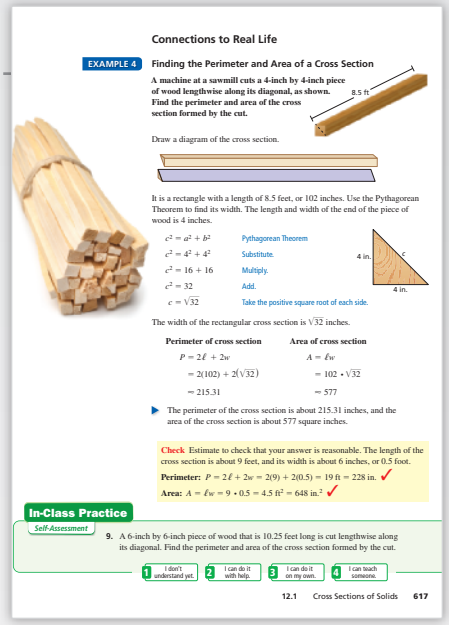
Entering–Emerging: Assist students with drawing a diagram and writing equations. Provide a sentence frame to guide students with stating their answers. Have partners practice reading their sentence aloud.

Developing–Expanding: Assist students with drawing a diagram and writing equations. Support students with using the wording in the problem to form a sentence explaining the context of their answers. After partners compare sentences, ask, "How can you add details to your sentence?"

Bridging–Reaching: Have students write a sentence to explain their answers. Then have students compare sentences with a partner. "How can your partner add to their sentence to provide more details?"

Talk About It

"Reflect on the last success criterion. Do you need additional examples or support to help you understand how to use cross sections to solve real-life problems?"



Connections to Real Life

EXAMPLE 4 Finding the Perimeter and Area of a Cross Section

A machine at a sawmill cuts a 4-inch by 4-inch piece of wood lengthwise along its diagonal, as shown. Find the perimeter and area of the cross section formed by the cut.

Draw a diagram of the cross section.

It is a rectangle with a length of 8.5 feet, or 102 inches. Use the Pythagorean Theorem to find its width. The length and width of the end of the piece of wood is 4 inches.

$c^2 = a^2 + b^2$ Pythagorean Theorem
$c^2 = 4^2 + 4^2$ Substitute.
$c^2 = 16 + 16$ Multiply.
$c^2 = 32$ Add.
$c = \sqrt{32}$ Take the positive square root of each side.

The width of the rectangular cross section is $\sqrt{32}$ inches.

Perimeter of cross section	Area of cross section
$P = 2\ell + 2w$	$A = \ell w$
$= 2(102) + 2(\sqrt{32})$	$= 102 \cdot \sqrt{32}$
≈ 215.31	≈ 577

▶ The perimeter of the cross section is about 215.31 inches, and the area of the cross section is about 577 square inches.

Check Estimate to check that your answer is reasonable. The length of the cross section is about 9 feet, and its width is about 6 inches, or 0.5 foot.
Perimeter: $P = 2\ell + 2w = 2(9) + 2(0.5) = 19$ ft $= 228$ in. ✓
Area: $A = \ell w = 9 \cdot 0.5 = 4.5$ ft$^2 = 648$ in.^{2} ✓

In-Class Practice

Self-Assessment

9. A 6-inch by 6-inch piece of wood that is 10.25 feet long is cut lengthwise along its diagonal. Find the perimeter and area of the cross section formed by the cut.

1 I don't understand yet. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone.

12.1 Cross Sections of Solids 617

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Quick Check Exercise 9

Paul's Notes

Closure

- **Exit Ticket:** "Sketch and name the different shapes that can be formed by cross sections of a cube." Check students' sketches; triangle, square, rectangle, rhombus, parallelogram, trapezoid, pentagon, and hexagon
- See *Support for All Learners* for content support to nudge students toward mastery.

My Thoughts on the Lesson

Go online to reflect on this lesson. How did you support student learning? What worked? What didn't work? What would you do differently?

12.1 Support for All Learners

Supporting Student Learning

Use the **Quick Check** exercises to assess understanding of lesson concepts and support emerging learners with additional resources. Remember to check students' Self-Assessment to help inform your instructional decisions.

Reinforce (TIER 1)

Identify where students need support with *this lesson*.

- **Exercise 1:** Classify solids.
- **Exercise 7:** Describe and draw cross sections.
- **Exercise 9:** Solve real-life problems involving cross sections.

Re-engage (TIER 2)

- Classify and describe polygons. **Geometry Section 1.4**

Intervene (TIER 3)

Identify where students have additional *unfinished learning*.

- Find areas of parallelograms, triangles, trapezoids, kites, and composite figures. Solve mathematical and real-life problems involving area. **Grade 6 Sections 7.1–7.3**

Deepening Student Learning

Support proficient and advanced learners with suggested resources targeted to promote more complex thinking and deeper understanding of lesson concepts.

Differentiated Resources

Learning is individualized and students may move in and out of proficiency levels with each skill and concept. The online resources provide a wide range of layered support for the diverse needs in your classroom.

RESOURCES

India's Notes

Equity in Action

Students who want to soar academically may have self-doubt while working on challenging tasks in your class. If they lose confidence, they may feel disappointed in themselves and worry about their overall performance. Look for ways to encourage and inspire your students and remind them that they can succeed in your class.

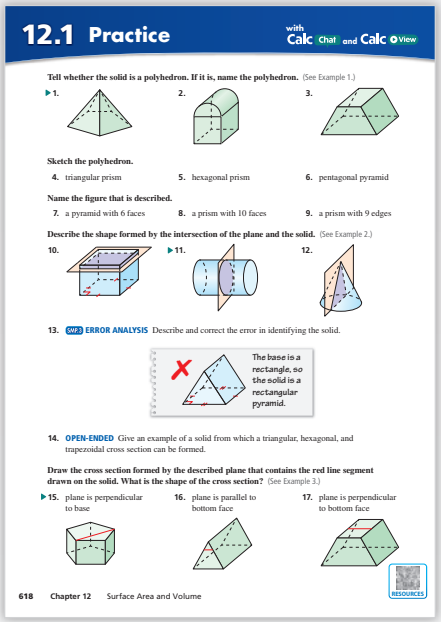
Listen to India talk about her insights and inspiration on equity in action.

12.1 Practice

See the *Answer Guide* for all Practice notes and answers to help inform next steps for your students.

Data Talk

See the *Answer Guide* for suggestions on how to connect the data display to math and facilitate classroom discussion.


12.1 Practice

with Calc Chat and Calc View

Tell whether the solid is a polyhedron. If it is, name the polyhedron. (See Example 1.)

1. 2. 3.

Sketch the polyhedron.

4. triangular prism 5. hexagonal prism 6. pentagonal pyramid

Name the figure that is described.

7. a pyramid with 6 faces 8. a prism with 10 faces 9. a prism with 9 edges

Describe the shape formed by the intersection of the plane and the solid. (See Example 2.)

10. 11. 12.

13. ERROR ANALYSIS Describe and correct the error in identifying the solid.

The base is a rectangle, so the solid is a rectangular pyramid.

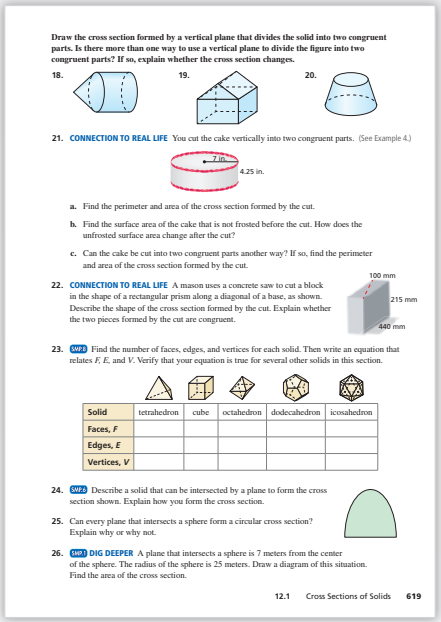
14. OPEN-ENDED Give an example of a solid from which a triangular, hexagonal, and trapezoidal cross section can be formed.

Draw the cross section formed by the described plane that contains the red line segment drawn on the solid. What is the shape of the cross section? (See Example 3.)

15. plane is perpendicular to base 16. plane is parallel to bottom face 17. plane is perpendicular to bottom face

RESOURCES

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Draw the cross section formed by a vertical plane that divides the solid into two congruent parts. Is there more than one way to use a vertical plane to divide the figure into two congruent parts? If so, explain whether the cross section changes.

18. 19. 20.

21. CONNECTION TO REAL LIFE You cut the cake vertically into two congruent parts. (See Example 4.)

7 in. 4.25 in.

a. Find the perimeter and area of the cross section formed by the cut.

b. Find the surface area of the cake that is not frosted before the cut. How does the unfrosted surface area change after the cut?

c. Can the cake be cut into two congruent parts another way? If so, find the perimeter and area of the cross section formed by the cut.

22. CONNECTION TO REAL LIFE A mason uses a concrete saw to cut a block in the shape of a rectangular prism along a diagonal of a base, as shown. Describe the shape of the cross section formed by the cut. Explain whether the two pieces formed by the cut are congruent.

100 mm 215 mm 440 mm

23. MP 8 Find the number of faces, edges, and vertices for each solid. Then write an equation that relates F, E, and V. Verify that your equation is true for several other solids in this section.

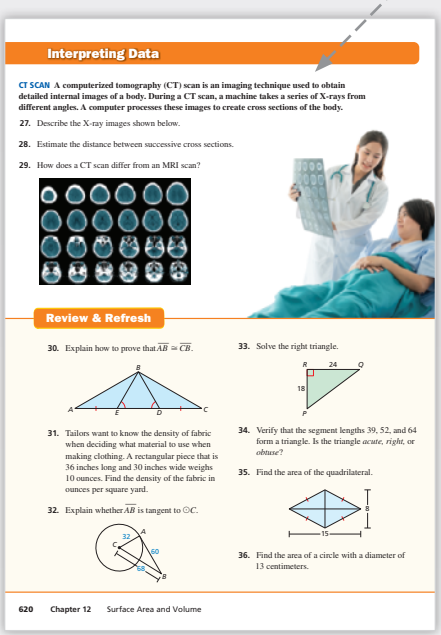
Solid	tetrahedron	cube	octahedron	dodecahedron	icosahedron
Faces, F					
Edges, E					
Vertices, V					

24. MP 2 Describe a solid that can be intersected by a plane to form the cross section shown. Explain how you form the cross section.

25. Can every plane that intersects a sphere form a circular cross section? Explain why or why not.

26. MP 3 DIG DEEPER A plane that intersects a sphere is 7 meters from the center of the sphere. The radius of the sphere is 25 meters. Draw a diagram of this situation. Find the area of the cross section.

12.1 Cross Sections of Solids 619


Interpreting Data

CT SCAN A computerized tomography (CT) scan is an imaging technique used to obtain detailed internal images of a body. During a CT scan, a machine takes a series of X-rays from different angles. A computer processes these images to create cross sections of the body.

27. Describe the X-ray images shown below.

28. Estimate the distance between successive cross sections.

29. How does a CT scan differ from an MRI scan?

Review & Refresh

30. Explain how to prove that $\overline{AB} \cong \overline{CB}$.

31. Tailors want to know the density of fabric when deciding what material to use when making clothing. A rectangular piece that is 36 inches long and 30 inches wide weighs 10 ounces. Find the density of the fabric in ounces per square yard.

32. Explain whether $\overline{AB}$ is tangent to $\odot C$.

33. Solve the right triangle.

34. Verify that the segment lengths 39, 52, and 64 form a triangle. Is the triangle *acute*, *right*, or *obtuse*?

35. Find the area of the quadrilateral.

36. Find the area of a circle with a diameter of 13 centimeters.

620 Chapter 12 Surface Area and Volume

Student Edition pages 618–620

12.2 Volumes of Prisms and Cylinders

Learning Target:
Find and use volumes of prisms and cylinders.

Success Criteria:
- Find volumes of prisms and cylinders.
- Solve real-life problems involving volumes of prisms and cylinders.
- Find surface areas and volumes of similar solids.

Prerequisite Skills
- Find areas of regular polygons. **Geometry Section 11.3**
- Use the formula for the area of a circle to find measures. **Geometry Section 11.2**
- Find areas of parallelograms, triangles, and composite figures. Solve mathematical and real-life problems involving area. **Grade 6 Sections 7.1 and 7.2**

Content Standards for Mathematics
- Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone.
- Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.

WIDA English Language Development Standards
ELD-MA.9-12.Explain.Interpretive Interpret mathematical explanations by identifying a concept or an entity

ELD-MA.9-12.Argue.Expressive Construct mathematics arguments that generalize logical relationships across cases*

*See the *Answer Guide.*

Paul's Notes

Launch
Goal: Students will use a real-life context to explore the volumes of a cylinder and a rectangular prism.

- Bring in a can of food and a small food storage container that is in the shape of a rectangular prism. The objects should be very close in volume so that students cannot easily determine whether the contents of the can will fit inside the container. Hold up the two objects but keep them separated.

? "Do you think the contents of this can will fit inside this container? Explain."

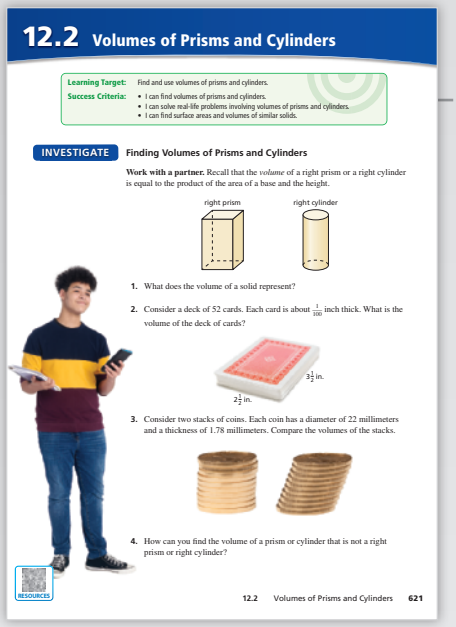
Talk About It
◎ "This question can be answered if you know the volume of each object."

Formative Assessment Tip
Rating Scale
At the end of the first day of instruction for a section, post a Self-Assessment scale on the wall for each class.

1	2	3	4
I don't understand yet.	I can do it with help.	I can do it on my own.	I can teach someone.

As students leave your room, give each student a sticker and tell them to place the sticker on their level of understanding. At the end of the last day of instruction, display a new copy of the Self-Assessment scale. Have students place stickers on the scale to rate their current understanding of the material. Compare the two rating scales to check for growth.

Use **Quick Check** exercises to quickly identify where students are in their learning.



12.2 Volumes of Prisms and Cylinders

Learning Target: Find and use volumes of prisms and cylinders.

Success Criteria:
- I can find volumes of prisms and cylinders.
- I can solve real-life problems involving volumes of prisms and cylinders.
- I can find surface areas and volumes of similar solids.

INVESTIGATE Finding Volumes of Prisms and Cylinders

Work with a partner. Recall that the *volume* of a right prism or a right cylinder is equal to the product of the area of a base and the height.

right prism right cylinder

1. What does the volume of a solid represent?
2. Consider a deck of 52 cards. Each card is about $\frac{1}{100}$ inch thick. What is the volume of the deck of cards?

$3\frac{1}{2}$ in. $2\frac{1}{2}$ in.

3. Consider two stacks of coins. Each coin has a diameter of 22 millimeters and a thickness of 1.78 millimeters. Compare the volumes of the stacks.
4. How can you find the volume of a prism or cylinder that is not a right prism or right cylinder?

RESOURCES

12.2 Volumes of Prisms and Cylinders 621

Student Edition page 621

Paul's Notes

Investigate

Goal: Students will find volumes of prisms and cylinders.

- Students may recall the formulas for the volume of a prism and the volume of a cylinder, but do not provide them. Allow partners to make conjectures based on their discussions and prior experiences.
- Circulate and listen to students' strategies for finding the volume of the deck of cards and the volumes of the stacks of coins.

? "How do you find the volume of a right cylinder?" Students' explanations should include multiplying the area of the circular base by the height of the cylinder.

? "How does your process for finding the volume change when the stack of coins does not form a right cylinder?"

Talk About It

"In this lesson, you will review the formulas for the volume of a prism and the volume of a cylinder. Then you will apply the formulas to various prisms and cylinders, some of which are not right prisms nor right cylinders. You will also work with concepts related to similar solids."

Differentiating Instruction

- **Emerging:** Students will benefit from holding and manipulating physical models to examine attributes. Prisms that are not rectangular are particularly helpful. Make connections between the structures of prisms and cylinders by saying, "You are finding the area of the base and then multiplying by the height." Students should have a list of area formulas and a calculator. Students may not have experiences with the concepts related to similar solids. Use models of cubes when finding surface areas and volumes of similar solids.
- **Proficient:** Students can use their algebraic skills to successfully work with the formulas in the lesson. Students may not have experiences with the concepts related to similar solids. If necessary, use models of cubes when finding surface areas and volumes of similar solids.

Paul's Notes

Finding Volumes of Prisms and Cylinders

- Use physical models to help students visualize volumes of solids, particularly the solids shown at the top of the Student Edition page. State the Key Concept and emphasize that the variable B represents the area of the base.

? Pose Example 1 and ask, "What is the shape of the base of the prism?" a triangle "What formula can you use to find the area of the base?" $A = \frac{1}{2}bh$

EL English Learner Support

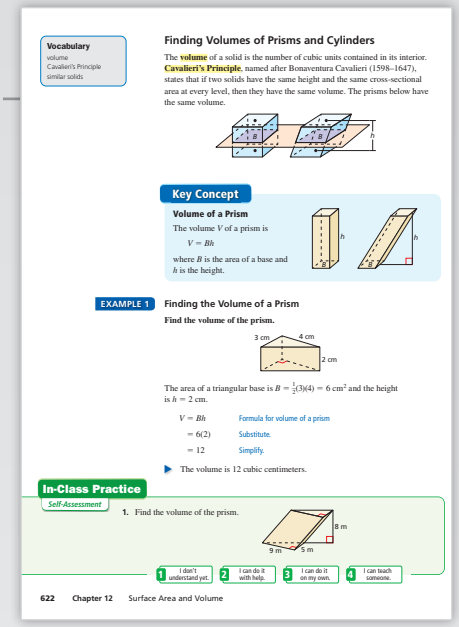
Interpreting Mathematical Explanations

Use images to review volumes of prisms and cylinders from previous courses. Support students with the In-Class Practice exercises in the following ways.

Entering–Emerging: Have partners work together to solve the exercises. Provide a word bank with examples for students to use to read their answers aloud. Have students form phrases or simple sentences and then point to each word as they read their answers aloud.

Developing–Expanding: Have partners work together to solve the exercises. Have students form their answers using complete sentences. Encourage partners to support each other's use of language.

Bridging–Reaching: Have students work independently to solve the exercises and then compare answers with a partner. Ensure students use precise mathematical terms as they read their answers aloud.


Vocabulary
volume
Cavalieri's Principle
similar solids

Finding Volumes of Prisms and Cylinders

The **volume** of a solid is the number of cubic units contained in its interior. **Cavalieri's Principle**, named after Bonaventura Cavalieri (1598–1647), states that if two solids have the same height and the same cross-sectional area at every level, then they have the same volume. The prisms below have the same volume.

Key Concept

Volume of a Prism

The volume V of a prism is

$V = Bh$

where B is the area of a base and h is the height.

EXAMPLE 1 Finding the Volume of a Prism

Find the volume of the prism.

The area of a triangular base is $B = \frac{1}{2}(3)(4) = 6$ cm^2 and the height is $h = 2$ cm.

$V = Bh$ Formula for volume of a prism

$= 6(2)$ Substitute.

$= 12$ Simplify.

▶ The volume is 12 cubic centimeters.

In-Class Practice

Self-Assessment

1. Find the volume of the prism.

1 I don't understand yet. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone.

622 Chapter 12 Surface Area and Volume

Student Edition page 622

Quick Check Exercise 1

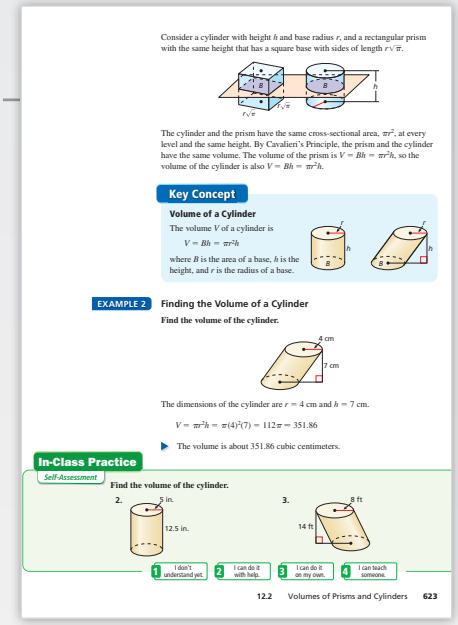
Paul's Notes

Finding Volumes of Prisms and Cylinders

- Remind students that Cavalieri's Principle applies to any type of solid, not just prisms. Point out the comparison of the volumes of a prism and a cylinder.
- State the Key Concept and have students work through Example 2 independently before comparing answers with a neighbor. Make sure students included cubic centimeters in their answer.

Talk About It

◎ "Are you confident that you can find the volume of a prism or a cylinder?"


Consider a cylinder with height h and base radius r, and a rectangular prism with the same height that has a square base with sides of length $r\sqrt{\pi}$.

The cylinder and the prism have the same cross-sectional area, πr^2, at every level and the same height. By Cavalieri's Principle, the prism and the cylinder have the same volume. The volume of the prism is $V = Bh = \pi r^2 h$, so the volume of the cylinder is also $V = Bh = \pi r^2 h$.

Key Concept

Volume of a Cylinder

The volume V of a cylinder is

$V = Bh = \pi r^2 h$

where B is the area of a base, h is the height, and r is the radius of a base.

EXAMPLE 2 Finding the Volume of a Cylinder

Find the volume of the cylinder.

The dimensions of the cylinder are $r = 4$ cm and $h = 7$ cm.

$V = \pi r^2 h = \pi(4)^2(7) = 112\pi \approx 351.86$

▶ The volume is about 351.86 cubic centimeters.

In-Class Practice

Self-Assessment

Find the volume of the cylinder.

2. 5 in., 12.5 in.

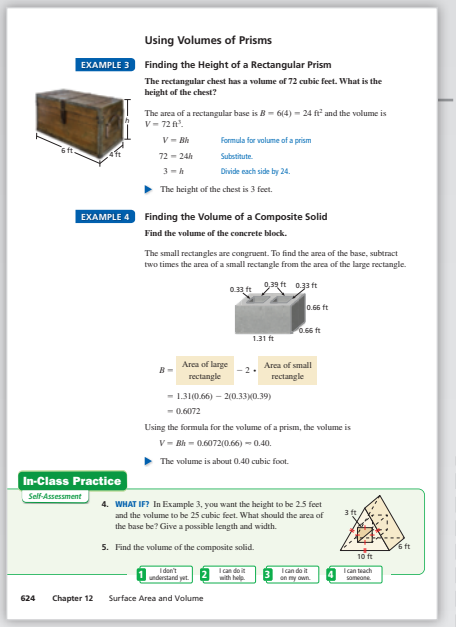
3. 8 ft, 14 ft

1 I don't understand yet. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone.

12.2 Volumes of Prisms and Cylinders 623

Student Edition page 623

Quick Check Exercise 2



Using Volumes of Prisms

EXAMPLE 3 Finding the Height of a Rectangular Prism

The rectangular chest has a volume of 72 cubic feet. What is the height of the chest?

The area of a rectangular base is $B = 6(4) = 24$ ft^2 and the volume is $V = 72$ ft^3.

$V = Bh$ Formula for volume of a prism

$72 = 24h$ Substitute.

$3 = h$ Divide each side by 24.

▶ The height of the chest is 3 feet.

EXAMPLE 4 Finding the Volume of a Composite Solid

Find the volume of the concrete block.

The small rectangles are congruent. To find the area of the base, subtract two times the area of a small rectangle from the area of the large rectangle.

$B =$ Area of large rectangle $- 2 \cdot$ Area of small rectangle

$= 1.31(0.66) - 2(0.33)(0.39)$

$= 0.6072$

Using the formula for the volume of a prism, the volume is

$V = Bh = 0.6072(0.66) \approx 0.40$.

▶ The volume is about 0.40 cubic foot.

In-Class Practice

Self-Assessment

4. **WHAT IF?** In Example 3, you want the height to be 2.5 feet and the volume to be 25 cubic feet. What should the area of the base be? Give a possible length and width.

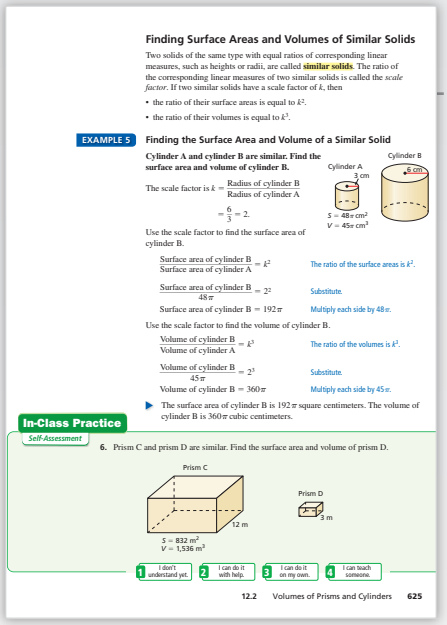
5. Find the volume of the composite solid.

1 I don't understand yet. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone.

624 Chapter 12 Surface Area and Volume

Student Edition page 624

Quick Check Exercise 4



Finding Surface Areas and Volumes of Similar Solids

Two solids of the same type with equal ratios of corresponding linear measures, such as heights or radii, are called **similar solids**. The ratio of the corresponding linear measures of two similar solids is called the *scale factor*. If two similar solids have a scale factor of k, then

- the ratio of their surface areas is equal to k^2.
- the ratio of their volumes is equal to k^3.

EXAMPLE 5 Finding the Surface Area and Volume of a Similar Solid

Cylinder A and cylinder B are similar. Find the surface area and volume of cylinder B.

The scale factor is $k = \frac{\text{Radius of cylinder B}}{\text{Radius of cylinder A}} = \frac{6}{3} = 2$.

Use the scale factor to find the surface area of cylinder B.

$\frac{\text{Surface area of cylinder B}}{\text{Surface area of cylinder A}} = k^2$ The ratio of the surface areas is k^2.

$\frac{\text{Surface area of cylinder B}}{48\pi} = 2^2$ Substitute.

Surface area of cylinder B $= 192\pi$ Multiply each side by 48π.

Use the scale factor to find the volume of cylinder B.

$\frac{\text{Volume of cylinder B}}{\text{Volume of cylinder A}} = k^3$ The ratio of the volumes is k^3.

$\frac{\text{Volume of cylinder B}}{45\pi} = 2^3$ Substitute.

Volume of cylinder B $= 360\pi$ Multiply each side by 45π.

▶ The surface area of cylinder B is 192π square centimeters. The volume of cylinder B is 360π cubic centimeters.

In-Class Practice

Self-Assessment

6. Prism C and prism D are similar. Find the surface area and volume of prism D.

1 I don't understand yet. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone.

12.2 Volumes of Prisms and Cylinders 625

Student Edition page 625

Quick Check Exercise 6

Paul's Notes

Using Volumes of Prisms

- Have students work on Examples 3 and 4 with a partner. Then ask students to share their strategies and results with the class. Highlight different approaches so that students can see there can be more than one way to solve a problem.

? **Extension:** "If the chest in Example 3 had a length of 6 feet 2 inches and a width of 40 inches, would your method for solving the problem change? Explain." No, but the dimensions and volume must be written in terms of the same units.

Paul's Notes

Finding Surface Areas and Volumes of Similar Solids

- Discuss similar solids with students. Note the difference between the exponents in the ratios—the scale factor is squared for surface area and cubed for volume.

? "Where have you studied a similar concept?" Students should make connections to their previous work with dilations.

- Have students work on Example 5 with a partner and then share their results with the class.

Talk About It

"Where are you in your learning? Do you understand how to find surface areas and volumes of similar solids?"

Paul's Notes

Closure

- **Exit Ticket:** "The Leaning Tower of Pisa is approximately a cylinder that has a height of 55.9 meters and a diameter of 15.5 meters. Approximate its volume." about 10,547.88 m^3
- See *Support for All Learners* for content support to nudge students toward mastery.

My Thoughts on the Lesson

Go online to reflect on this lesson. How did you support student learning? What worked? What didn't work? What would you do differently?

12.2 Support for All Learners

Supporting Student Learning

Use the **Quick Check** exercises to assess understanding of lesson concepts and support emerging learners with additional resources. Remember to check students' Self-Assessment to help inform your instructional decisions.

Reinforce (TIER 1)

Identify where students need support with *this lesson*.

- **Exercises 1 and 2:** Find volumes of prisms and cylinders.
- **Exercise 4:** Solve real-life problems involving volumes of prisms and cylinders.
- **Exercise 6:** Find surface areas and volumes of similar solids.

Re-engage (TIER 2)

Identify where students need support with *prior skills*.

- Use the formula for the area of a circle to find measures. **Geometry Section 11.2**

Intervene (TIER 3)

Identify where students have additional *unfinished learning*.

- Find areas of parallelograms, triangles, trapezoids, kites, and composite figures. Solve mathematical and real-life problems involving area. **Grade 6 Sections 7.1–7.3**

Deepening Student Learning

Support proficient and advanced learners with suggested resources targeted to promote more complex thinking and deeper understanding of lesson concepts.

Differentiated Resources

Learning is individualized and students may move in and out of proficiency levels with each skill and concept. The online resources provide a wide range of layered support for the diverse needs in your classroom.

RESOURCES

India's Notes

Equity in Action

Some students may not fully understand a topic until a few days after the lesson was taught. This delayed understanding may take away from the pacing of the current lesson. Celebrate these exciting moments of understanding while continuing to move all students forward in their learning.

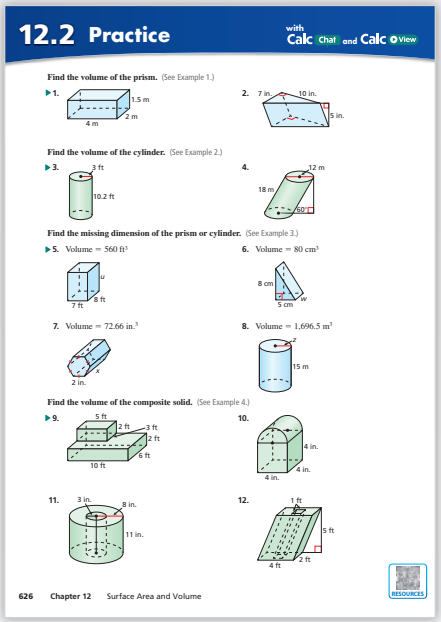
Listen to India talk about her insights and inspiration on equity in action.

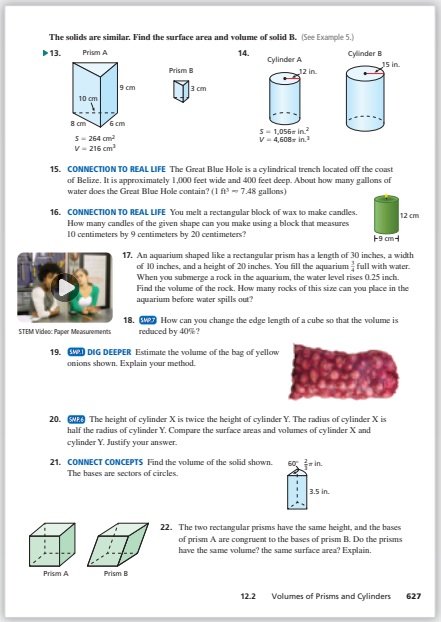
12.2 Practice

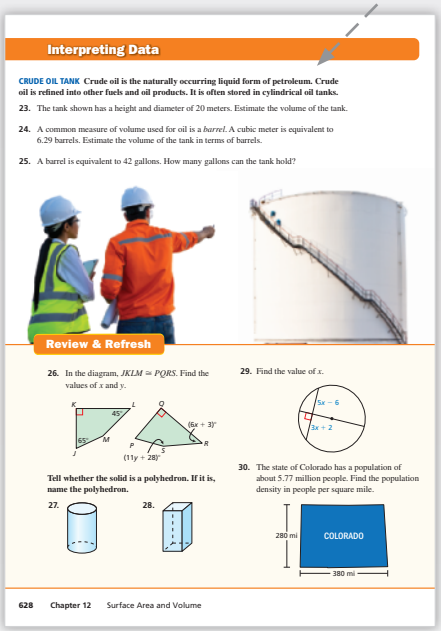
See the *Answer Guide* for all Practice notes and answers to help inform next steps for your students.

Data Talk

See the *Answer Guide* for suggestions on how to connect the data display to math and facilitate classroom discussion.







Student Edition pages 626–628

12.3 Volumes of Pyramids

Learning Target:
Find and use volumes of pyramids.

Success Criteria:
- Find volumes of pyramids.
- Use volumes of pyramids to find measures.
- Find volumes of similar pyramids.
- Find volumes of composite solids containing pyramids.

Prerequisite Skills
- Find volumes of prisms. **Geometry Section 12.2**
- Find areas of regular polygons. **Geometry Section 11.3**

Content Standards for Mathematics
- Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone.
- Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.

WIDA English Language Development Standards
ELD-MA.9-12.Explain.Expressive Construct mathematical explanations that share solutions with others

ELD-MA.9-12.Explain.Interpretive Interpret mathematical explanations by analyzing data and owning problem-solving approaches*

*See the *Answer Guide*.

Paul's Notes

Launch

Goal: Students will explore famous pyramids in Egypt to spark interest about finding volumes of pyramids.

- Display pictures of the Giza Pyramids in Egypt. Explain that the Great Pyramid of Giza is one of the Seven Wonders of the Ancient World.
- Show students a picture of the Great Pyramid.
- "The Great Pyramid consists of about 2.3 million stone blocks, each weighing about 2.5 to 15 tons. It originally stood about 482 feet high, and its base had four equal side lengths that each measured about 756 feet."

? "How would you classify the Great Pyramid?" square pyramid

Talk About It

"In this section, you will find volumes of pyramids."

Use **Quick Check** exercises to quickly identify where students are in their learning.

Paul's Notes

Investigate

Goal: Students will derive a formula for the volume of a pyramid.

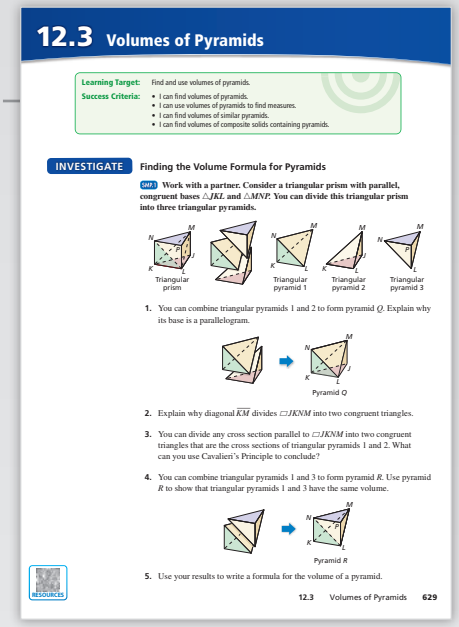
- Consider providing a physical model for students to explore as they work through this Investigate. Alternatively, students can use dynamic geometry software to rotate and analyze a pyramid.
- In Exercise 3, students should recognize that the triangular pyramids have the same volume.
- This Investigate can be highly abstract for students. Do not spend too much time dwelling on any lack of progress you observe. If necessary, proceed directly to the formal lesson.

Talk About It

"In the lesson, you will apply the formula for the volume of a pyramid. You will also work with similar pyramids and with composite solids that contain pyramids."

Differentiating Instruction

Emerging: Student will benefit from holding and manipulating physical models to examine attributes. Alternatively, they can use dynamic geometry software to manipulate and examine various pyramids. Watch for students who incorrectly multiply $\frac{1}{3}$ when using the formula for the volume of a pyramid. They may mistakenly "distribute" this constant to more than one value in the formula.



12.3 Volumes of Pyramids

Learning Target: Find and use volumes of pyramids.

Success Criteria:
- I can find volumes of pyramids.
- I can use volumes of pyramids to find measures.
- I can find volumes of similar pyramids.
- I can find volumes of composite solids containing pyramids.

INVESTIGATE Finding the Volume Formula for Pyramids

DYI Work with a partner. Consider a triangular prism with parallel, congruent bases $\triangle JKL$ and $\triangle MNP$. You can divide this triangular prism into three triangular pyramids.

Triangular prism | Triangular pyramid 1 | Triangular pyramid 2 | Triangular pyramid 3

1. You can combine triangular pyramids 1 and 2 to form pyramid Q. Explain why its base is a parallelogram.

Pyramid Q

2. Explain why diagonal $\overline{KM}$ divides $\square JKNM$ into two congruent triangles.

3. You can divide any cross section parallel to $\square JKNM$ into two congruent triangles that are the cross sections of triangular pyramids 1 and 2. What can you use Cavalieri's Principle to conclude?

4. You can combine triangular pyramids 1 and 3 to form pyramid R. Use pyramid R to show that triangular pyramids 1 and 3 have the same volume.

Pyramid R

5. Use your results to write a formula for the volume of a pyramid.

RESOURCES

12.3 Volumes of Pyramids 629

Student Edition page 629

Paul's Notes

Finding Volumes of Pyramids

- State the formula for the volume of a pyramid in the Key Concept.

? "How can you find the area of the base of a pyramid?" Students should recognize that it depends on the shape of the base.

- Have students complete Example 1 independently and then compare answers with a neighbor.

EL English Learner Support

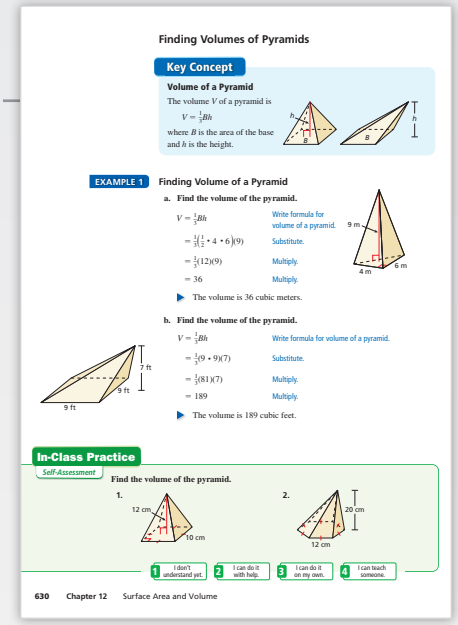
Constructing Mathematical Explanations

Have EL students add the Key Concept to their math vocabulary notebooks. Support students with Exercises 1 and 2 by having students use sequential connectors (first, then) to explain their solution steps.

Entering–Emerging: Provide students with a word bank of sequential connectors to use to label their solution steps. Then have them share their steps with a partner.

Developing–Expanding: Provide sentence starters for students to use to explain their solution steps. For example, "First, I _____. Then, I _____. My last step was _____." Students may point to each solution step as they read their sentences aloud to a partner.

Bridging–Reaching: Have students discuss their solution steps using sequential connectors and complete sentences. Have students refer to Example 1 if they need assistance with wording their sentences.



Finding Volumes of Pyramids

Key Concept

Volume of a Pyramid

The volume V of a pyramid is $V = \frac{1}{3}Bh$ where B is the area of the base and h is the height.

EXAMPLE 1 Finding Volume of a Pyramid

a. Find the volume of the pyramid.

$V = \frac{1}{3}Bh$ Write formula for volume of a pyramid.
$= \frac{1}{3}(\frac{1}{2} \cdot 4 \cdot 6)(9)$ Substitute.
$= \frac{1}{3}(12)(9)$ Multiply.
$= 36$ Multiply.

The volume is 36 cubic meters.

b. Find the volume of the pyramid.

$V = \frac{1}{3}Bh$ Write formula for volume of a pyramid.
$= \frac{1}{3}(9 \cdot 9)(7)$ Substitute.
$= \frac{1}{3}(81)(7)$ Multiply.
$= 189$ Multiply.

The volume is 189 cubic feet.

In-Class Practice

Self-Assessment

Find the volume of the pyramid.

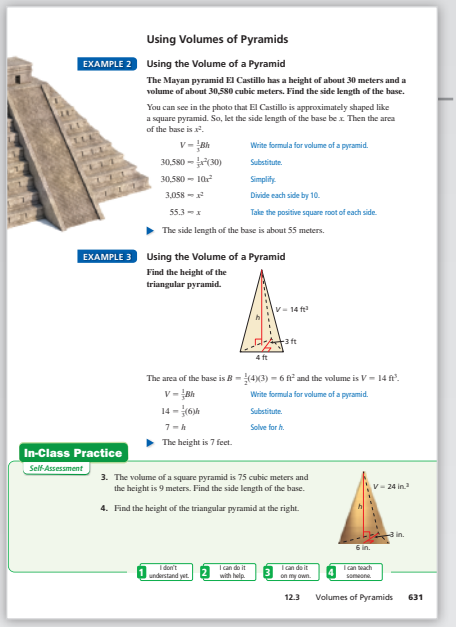
1. 2.

1 I don't understand yet. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone.

630 Chapter 12 Surface Area and Volume

Student Edition page 630

Quick Check Exercise 1



Using Volumes of Pyramids

EXAMPLE 2 Using the Volume of a Pyramid

The Mayan pyramid El Castillo has a height of about 30 meters and a volume of about 30,580 cubic meters. Find the side length of the base.

You can see in the photo that El Castillo is approximately shaped like a square pyramid. So, let the side length of the base be x. Then the area of the base is x^2.

$V = \frac{1}{3}Bh$ Write formula for volume of a pyramid.

$30{,}580 = \frac{1}{3}x^2(30)$ Substitute.

$30{,}580 = 10x^2$ Simplify.

$3{,}058 = x^2$ Divide each side by 10.

$55.3 \approx x$ Take the positive square root of each side.

▶ The side length of the base is about 55 meters.

EXAMPLE 3 Using the Volume of a Pyramid

Find the height of the triangular pyramid.

$V = 14$ ft³, h, 3 ft, 4 ft

The area of the base is $B = \frac{1}{2}(4)(3) = 6$ ft² and the volume is $V = 14$ ft³.

$V = \frac{1}{3}Bh$ Write formula for volume of a pyramid.

$14 = \frac{1}{3}(6)h$ Substitute.

$7 = h$ Solve for h.

▶ The height is 7 feet.

In-Class Practice

Self-Assessment

3. The volume of a square pyramid is 75 cubic meters and the height is 9 meters. Find the side length of the base.

4. Find the height of the triangular pyramid at the right.

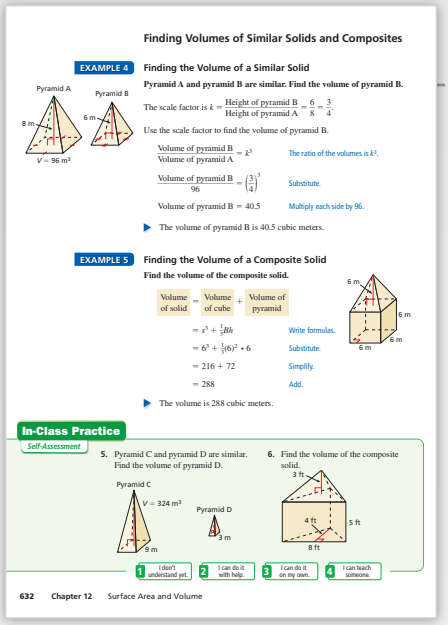
$V = 24$ in.³, h, 3 in., 6 in.

1 I don't understand yet. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone.

12.3 Volumes of Pyramids 631

Student Edition page 631

Quick Check Exercise 3



Finding Volumes of Similar Solids and Composites

EXAMPLE 4 Finding the Volume of a Similar Solid

Pyramid A and pyramid B are similar. Find the volume of pyramid B.

Pyramid A, Pyramid B, 8 m, 6 m, $V = 96$ m³

The scale factor is $k = \frac{\text{Height of pyramid B}}{\text{Height of pyramid A}} = \frac{6}{8} = \frac{3}{4}$.

Use the scale factor to find the volume of pyramid B.

$\frac{\text{Volume of pyramid B}}{\text{Volume of pyramid A}} = k^3$ The ratio of the volumes is k^3.

$\frac{\text{Volume of pyramid B}}{96} = \left(\frac{3}{4}\right)^3$ Substitute.

Volume of pyramid B $= 40.5$ Multiply each side by 96.

▶ The volume of pyramid B is 40.5 cubic meters.

EXAMPLE 5 Finding the Volume of a Composite Solid

Find the volume of the composite solid.

6 m, 6 m, 6 m

Volume of solid = Volume of cube + Volume of pyramid

$= s^3 + \frac{1}{3}Bh$ Write formulas.

$= 6^3 + \frac{1}{3}(6)^2 \cdot 6$ Substitute.

$= 216 + 72$ Simplify.

$= 288$ Add.

▶ The volume is 288 cubic meters.

In-Class Practice

Self-Assessment

5. Pyramid C and pyramid D are similar. Find the volume of pyramid D.

Pyramid C, $V = 324$ m³, 9 m, Pyramid D, 3 m

6. Find the volume of the composite solid.

3 ft, 4 ft, 5 ft, 8 ft

1 I don't understand yet. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone.

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Quick Check Exercises 5 and 6

Paul's Notes

Using Volumes of Pyramids

- **Example 2:** Students are given the volume and the height of a pyramid. Then they are asked to find a missing dimension. Make sure students recognize that the base of the pyramid is a square and the side length of the base is unknown, so they must represent its area as the square of a variable.
- **Think-Pair-Share:** Allow time for students to work through Example 3 independently. Then have partners compare their results before sharing with the whole class.
- Have students complete Exercises 3 and 4 with a partner. Ask volunteers to show their work on the board and explain their approaches to the class.

Paul's Notes

Finding Volumes of Similar Solids and Composites

- Before working through Example 4, ask, "If two similar solids have a scale factor of k, then what is the ratio of their surface areas? the ratio of their volumes?" k^2; k^3
- Work through Example 4 as shown.
- **Think-Pair-Share:** "What solids make up the composite solid in Example 5? How can you find the volume of the composite solid?"
- **Turn and Talk:** "If the principal were to come into our room right now, how would you explain how to find the volume of a composite solid? After discussing with your partner, write a summary of what you would say."

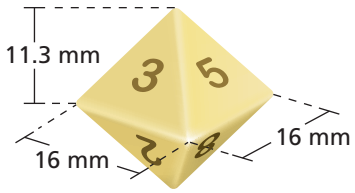
Talk About It

"Reflect on the last two success criteria. Where are you in your learning?"

Paul's Notes

Closure

- **Exit Ticket:** Have students find the volume of the octahedral die. about 1,928.5 mm³
- See *Support for All Learners* for content support to nudge students toward mastery.



My Thoughts on the Lesson

Go online to reflect on this lesson. How did you support student learning? What worked? What didn't work? What would you do differently?

12.3 Support for All Learners

Supporting Student Learning

Use the **Quick Check** exercises to assess understanding of lesson concepts and support emerging learners with additional resources. Remember to check students' Self-Assessment to help inform your instructional decisions.

Reinforce (TIER 1)

Identify where students need support with *this lesson*.

- **Exercise 1:** Find volumes of pyramids.
- **Exercise 3:** Use volumes of pyramids to find measures.
- **Exercise 5:** Find volumes of similar pyramids.
- **Exercise 6:** Find volumes of composite solids containing pyramids.

Re-engage (TIER 2)

Identify where students need support with *prior skills*.

- Find volumes of prisms. **Geometry Section 12.2**

Intervene (TIER 3)

Identify where students have additional *unfinished learning*.

- Find areas of regular polygons. **Geometry Section 11.3**

Deepening Student Learning

Support proficient and advanced learners with suggested resources targeted to promote more complex thinking and deeper understanding of lesson concepts.

Differentiated Resources

Learning is individualized and students may move in and out of proficiency levels with each skill and concept. The online resources provide a wide range of layered support for the diverse needs in your classroom.

RESOURCES

India's Notes

Equity in Action

Some students are so addicted to technology that they cannot focus during your class. They might try to use their electronic devices to watch videos or play games that are not related to your class. Help your students balance their time on electronics by being clear about the expectations of technology use in your classroom and helping students stay focused on the lesson and assignments.

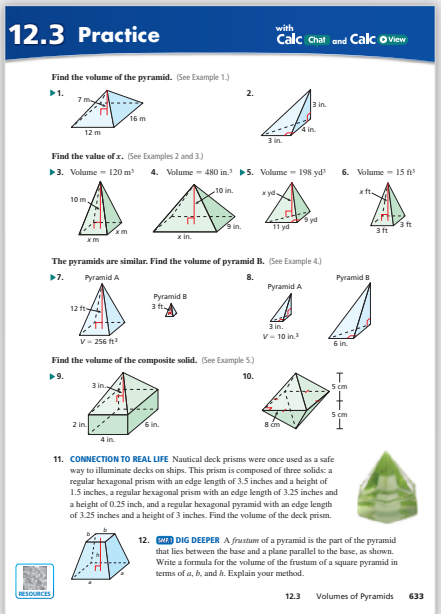
Listen to India talk about her insights and inspiration on equity in action.

12.3 Practice

See the *Answer Guide* for all Practice notes and answers to help inform next steps for your students.

Data Talk

See the *Answer Guide* for suggestions on how to connect the data display to math and facilitate classroom discussion.



12.3 Practice

with Calc Chat and Calc View

Find the volume of the pyramid. (See Example 1.)

1. 7 m, 12 m, 16 m
2. 3 in., 3 in., 4 in.

Find the value of x. (See Examples 2 and 3.)

3. Volume = 120 m³: 10 m, x m
4. Volume = 480 in.³: 10 in., x in., 5 in.
5. Volume = 198 yd³: x yd, 11 yd, 9 yd
6. Volume = 15 ft³: x ft, 3 ft, 5 ft

The pyramids are similar. Find the volume of pyramid B. (See Example 4.)

7. Pyramid A: 12 ft, V = 256 ft³; Pyramid B: 3 ft
8. Pyramid A: 3 in., V = 10 in.³; Pyramid B: 6 in.

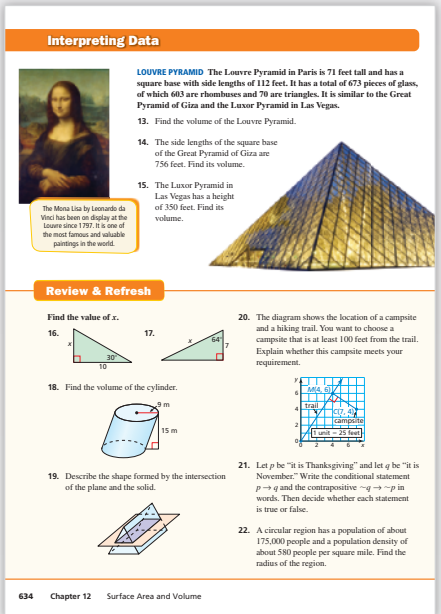
Find the volume of the composite solid. (See Example 5.)

9. 3 in., 2 in., 6 in., 4 in.
10. 5 cm, 5 cm, 8 cm

11. **CONNECTION TO REAL LIFE** Nautical deck prisms were once used as a safe way to illuminate decks on ships. This prism is composed of three solids: a regular hexagonal prism with an edge length of 3.5 inches and a height of 1.5 inches, a regular hexagonal prism with an edge length of 3.25 inches and a height of 0.25 inch, and a regular hexagonal pyramid with an edge length of 3.25 inches and a height of 3 inches. Find the volume of the deck prism.

12. **DIG DEEPER** A *frustum* of a pyramid is the part of the pyramid that lies between the base and a plane parallel to the base, as shown. Write a formula for the volume of the frustum of a square pyramid in terms of a, b, and h. Explain your method.

12.3 Volumes of Pyramids 633



Interpreting Data

LOUVRE PYRAMID The Louvre Pyramid in Paris is 71 feet tall and has a square base with side lengths of 112 feet. It has a total of 673 pieces of glass, of which 603 are rhombuses and 70 are triangles. It is similar to the Great Pyramid of Giza and the Luxor Pyramid in Las Vegas.

The Mona Lisa by Leonardo da Vinci has been on display at the Louvre since 1797. It is one of the most famous and valuable paintings in the world.

13. Find the volume of the Louvre Pyramid.
14. The side lengths of the square base of the Great Pyramid of Giza are 756 feet. Find its volume.
15. The Luxor Pyramid in Las Vegas has a height of 350 feet. Find its volume.

Review & Refresh

Find the value of x.

16. x, 30°, 10
17. x, 64, 7
18. Find the volume of the cylinder. 9 m, 15 m
19. Describe the shape formed by the intersection of the plane and the solid.
20. The diagram shows the location of a campsite and a hiking trail. You want to choose a campsite that is at least 100 feet from the trail. Explain whether this campsite meets your requirement. M(4, 6), trail, C(7, 4), campsite, 1 unit = 25 feet
21. Let p be "it is Thanksgiving" and let q be "it is November." Write the conditional statement $p \rightarrow q$ and the contrapositive $\sim q \rightarrow \sim p$ in words. Then decide whether each statement is true or false.
22. A circular region has a population of about 175,000 people and a population density of about 580 people per square mile. Find the radius of the region.

634 Chapter 12 Surface Area and Volume

Student Edition pages 633 and 634

12.4 Surface Areas and Volumes of Cones

Learning Target:
Find and use surface areas and volumes of cones.

Success Criteria:
- Find surface areas of cones.
- Find volumes of cones.
- Find the volumes of similar cones.
- Find the volumes of composite solids containing cones.

Prerequisite Skills
- Find volumes of cylinders. **Geometry Section 12.2**
- Use the formulas for the circumference and area of a circle to find measures. **Geometry Sections 11.1 and 11.2**
- Use the Pythagorean Theorem to find missing side lengths of right triangles. **Geometry Section 9.1**

Content Standards for Mathematics
- Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone.
- Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.

WIDA English Language Development Standards
ELD-MA.9-12.Explain.Expressive Construct mathematical explanations that state reasoning used to generate own or alternate solutions

ELD-MA.9-12.Explain.Expressive Construct mathematical explanations that introduce a mathematical concept or entity

Paul's Notes

Launch

Goal: Students will sketch the net of a cone.
- "Sketch the net of a cone."
- Ask students to share their sketches. Students may draw the net of the lateral surface incorrectly.
- You can support this activity by bringing in sugar cones that have wrappers on them.

Talk About It
"In the Investigate, you will use the net of a cone to develop a formula for the surface area of a cone."

Use **Quick Check** exercises to quickly identify where students are in their learning.

Paul's Notes

Investigate

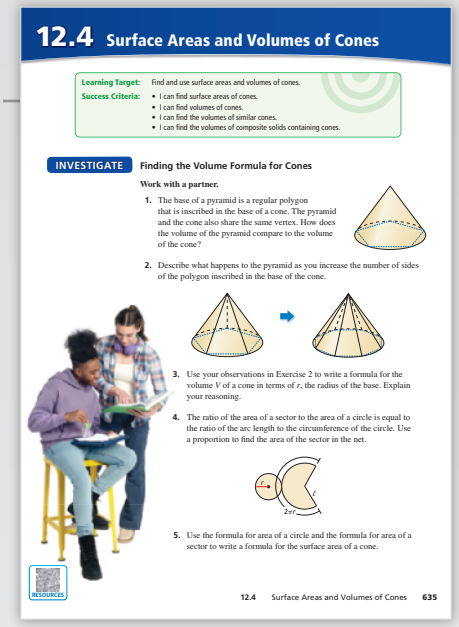
Goal: Students will derive formulas for the volume of a cone and the surface area of a cone.

- In Exercise 2, students should notice that as the number of sides of the polygon increases, the pyramid begins to look like a cone. Dynamic geometry software can help students visualize this. Some programs may have a slider for the number of sides of the polygon, which allows students to see this concept come to life.
- Allow ample time for partners to work on Exercises 4 and 5 before engaging in a class discussion. Investigating the lateral surface of a cone may be a new concept for students. Physical models that students are able to manipulate can be extremely helpful.

? "What does the net of a cone look like? How can you use the net to help you write a formula for the surface area of a cone?"

Talk About It

"In the lesson, you will apply the formulas for the volume and the surface area of a cone to solve problems. You will also work with similar cones and with composite solids that contain cones."



12.4 Surface Areas and Volumes of Cones

Learning Target: Find and use surface areas and volumes of cones.

Success Criteria:
- I can find surface areas of cones.
- I can find volumes of cones.
- I can find the volumes of similar cones.
- I can find the volumes of composite solids containing cones.

INVESTIGATE Finding the Volume Formula for Cones

Work with a partner.

1. The base of a pyramid is a regular polygon that is inscribed in the base of a cone. The pyramid and the cone also share the same vertex. How does the volume of the pyramid compare to the volume of the cone?
2. Describe what happens to the pyramid as you increase the number of sides of the polygon inscribed in the base of the cone.
3. Use your observations in Exercise 2 to write a formula for the volume V of a cone in terms of r, the radius of the base. Explain your reasoning.
4. The ratio of the area of a sector to the area of a circle is equal to the ratio of the arc length to the circumference of the circle. Use a proportion to find the area of the sector in the net.
5. Use the formula for area of a circle and the formula for area of a sector to write a formula for the surface area of a cone.

RESOURCES

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Making Math Visible

- If time permits, you can have students derive the formula for the surface area of a right cone using the following method.
- Begin by having students construct a circle with a radius of 3 inches. Have them mark the circumference of the circle as six equal parts and label the length of each part. Then they can cut out one sector of the circle and make a cone. Tell students to assume that the open circle represents the base of the cone. Have students record the radius of the base, slant height, area of the base, lateral area, and total surface area in a table.
- Have students continue removing sectors of the circle and recording the results.

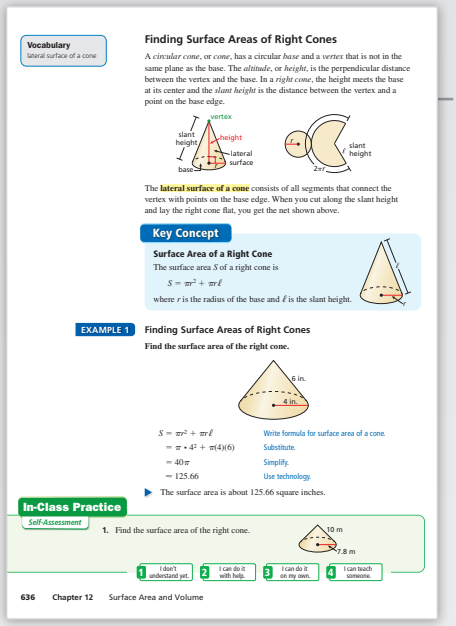


Lateral surface					
Base radius (in.)	$\frac{5}{2}$	2	$\frac{3}{2}$	1	$\frac{1}{2}$
Slant height (in.)	3	3	3	3	3
Base area (in.^{2})	$\pi\left(\frac{5}{2}\right)^2 = \frac{25}{4}\pi$	$\pi(2)^2 = 4\pi$	$\pi\left(\frac{3}{2}\right)^2 = \frac{9}{4}\pi$	$\pi(1)^2 = \pi$	$\pi\left(\frac{1}{2}\right)^2 = \frac{1}{4}\pi$
Lateral area (in.^{2})	$\frac{5}{6}\pi(3)^2 = \frac{15}{2}\pi$	$\frac{2}{3}\pi(3)^2 = 6\pi$	$\frac{1}{2}\pi(3)^2 = \frac{9}{2}\pi$	$\frac{1}{3}\pi(3)^2 = 3\pi$	$\frac{1}{6}\pi(3)^2 = \frac{3}{2}\pi$
Total surface area (in.^{2})	$\frac{55}{4}\pi$	10π	$\frac{27}{4}\pi$	4π	$\frac{7}{4}\pi$

- **Observations:** The radius of the base is decreasing. The radius of the lateral surface is always 3 inches, and it is also the slant height of the cone. To find the total surface area of the cone, you add the base area and the lateral area. Students will recognize that the area of the base is always πr^2. The pattern in the lateral area is that the coefficient of π is the radius of the base r times the slant height ℓ. So, the formula for the surface area of a cone is $S = \pi r^2 + r\ell\pi$, or $S = \pi r^2 + \pi r\ell$.

Differentiating Instruction

- **Emerging:** Students will benefit from holding and manipulating physical models to examine attributes. To help students see that the height of a right cone is less than its slant height, use a pencil to directly measure each dimension of a physical model. Remind students to leave expressions in terms of π until the last step of a calculation. Students will need calculators for this lesson.
- **Proficient:** Students can use their algebraic skills to successfully work with the formulas in the lesson. The formula for the surface area of a cone may be new to students. Have students work with a partner to review the Key Concepts and complete the In-Class Practice exercises.



Vocabulary
lateral surface of a cone

Finding Surface Areas of Right Cones

A *circular cone*, or *cone*, has a circular *base* and a *vertex* that is not in the same plane as the base. The *altitude*, or *height*, is the perpendicular distance between the vertex and the base. In a *right cone*, the height meets the base at its center and the *slant height* is the distance between the vertex and a point on the base edge.

vertex
slant height
height
lateral surface
base
slant height
$2\pi r$

The **lateral surface of a cone** consists of all segments that connect the vertex with points on the base edge. When you cut along the slant height and lay the right cone flat, you get the net shown above.

Key Concept

Surface Area of a Right Cone

The surface area S of a right cone is

$$S = \pi r^2 + \pi r\ell$$

where r is the radius of the base and ℓ is the slant height.

EXAMPLE 1 **Finding Surface Areas of Right Cones**

Find the surface area of the right cone.

6 in.
4 in.

$S = \pi r^2 + \pi r\ell$ Write formula for surface area of a cone.
$= \pi \cdot 4^2 + \pi(4)(6)$ Substitute.
$= 40\pi$ Simplify.
≈ 125.66 Use technology.

▶ The surface area is about 125.66 square inches.

In-Class Practice

Self-Assessment

1. Find the surface area of the right cone.

10 m
7.8 m

1 I don't understand yet. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone.

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Quick Check Exercise 1

Paul's Notes

Finding Surface Areas of Right Cones

- Spend time discussing the diagram of a cone and its two-dimensional net.
- **Fact-First Questioning:** "The height of a right cone must be less than its slant height. Why is this true?"
- Display and discuss the formula in the Key Concept.
- Pose Example 1 and allow time for students to work independently.

EL English Learner Support

Constructing Mathematical Explanations

Have EL students add the Key Concepts to their math vocabulary notebooks. Ensure students include color-coded images and examples. Support students with Exercises 1–3 in the following ways.

Entering–Emerging: Have partners complete Exercise 1. Then have them use sentence frames to explain their reasoning to another pair. "The radius of the base of the cone is _____ and the slant height is _____. So, the surface area of the cone is _____." Provide similar support for Exercises 2 and 3.

Developing–Expanding: Have partners complete Exercise 1. Then have them explain their reasoning to another pair. Have students discuss what they can add to their sentences to make their reasoning stronger. Provide similar support for Exercises 2 and 3.

Bridging–Reaching: Have students complete Exercise 1 and then write to explain their reasoning. Have students compare answers and reasoning with a partner. "What can you add to your sentences to make your reasoning stronger?" Provide similar support for Exercises 2 and 3.

Paul's Notes

Finding Volumes of Cones

- Display and discuss the formula for the volume of a cone. Students have already explored how increasing the number of sides of the base of a pyramid will cause the pyramid to approach a cone.
- Pose Example 2 and monitor students as they find the volume of the cone.
- Circulate and monitor as students complete Exercises 2 and 3. Look for common errors or misconceptions. Then use *Critique, Correct, and Clarify* to engage students in discussion.

Language Routine

Critique, Correct, and Clarify

This language routine provides an opportunity for students to reflect on a solution that is incomplete or incorrect. Strategically select and display a student work sample that is incomplete, incorrect, unclear, and/or demonstrates a common error. Do not disclose the author's name.

Ask students to analyze the work. Then have partners discuss the parts of the solution that were completed correctly, identify any errors, and co-craft a clear and correct solution. Discuss the corrected solutions as a whole class and encourage students to further refine their work.

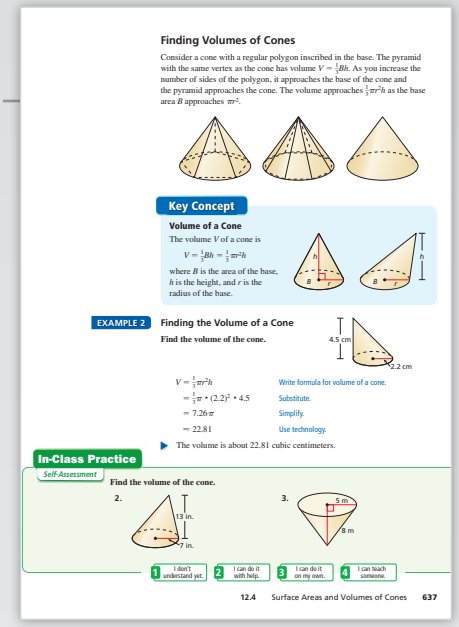
Talk About It

"Are you confident that you can find surface areas and volumes of cones? If not, do you need additional examples or support?"

India's Notes

Developing Grit

A student may come to class with low self-esteem or insecurities due to an experience that occurred outside of your classroom. You can build your student's confidence by showing that you care, celebrating effort, and praising perseverance. As you see your student start to come around, show that you care by saying, "I see you are feeling better. I know you have what it takes to be successful!" As you cultivate a classroom culture of safety and trust, encourage your students to recognize the grit they have deep within themselves that will help them show up and do their best work.



Finding Volumes of Cones

Consider a cone with a regular polygon inscribed in the base. The pyramid with the same vertex as the cone has volume $V = \frac{1}{3}Bh$. As you increase the number of sides of the polygon, it approaches the base of the cone and the pyramid approaches the cone. The volume approaches $\frac{1}{3}\pi r^2 h$ as the base area B approaches πr^2.

Key Concept

Volume of a Cone

The volume V of a cone is

$V = \frac{1}{3}Bh = \frac{1}{3}\pi r^2 h$

where B is the area of the base, h is the height, and r is the radius of the base.

EXAMPLE 2 Finding the Volume of a Cone

Find the volume of the cone.

4.5 cm, 2.2 cm

$V = \frac{1}{3}\pi r^2 h$ Write formula for volume of a cone.

$= \frac{1}{3}\pi \cdot (2.2)^2 \cdot 4.5$ Substitute.

$= 7.26\pi$ Simplify.

≈ 22.81 Use technology.

▶ The volume is about 22.81 cubic centimeters.

In-Class Practice

Self-Assessment

Find the volume of the cone.

2. 13 in., 7 in.

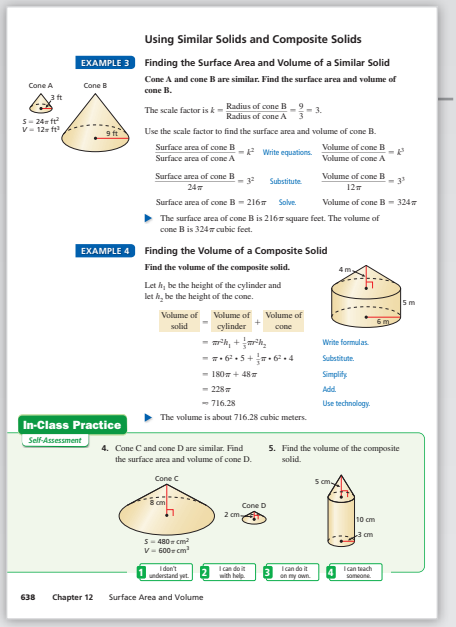
3. 5 m, 9 m

1 I don't understand yet. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone.

12.4 Surface Areas and Volumes of Cones 637

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Quick Check Exercise 3



Using Similar Solids and Composite Solids

EXAMPLE 3 Finding the Surface Area and Volume of a Similar Solid

Cone A and cone B are similar. Find the surface area and volume of cone B.

The scale factor is $k = \frac{\text{Radius of cone B}}{\text{Radius of cone A}} = \frac{9}{3} = 3$.

Use the scale factor to find the surface area and volume of cone B.

$\frac{\text{Surface area of cone B}}{\text{Surface area of cone A}} = k^2$ Write equations. $\frac{\text{Volume of cone B}}{\text{Volume of cone A}} = k^3$

$\frac{\text{Surface area of cone B}}{24\pi} = 3^2$ Substitute. $\frac{\text{Volume of cone B}}{12\pi} = 3^3$

Surface area of cone B $= 216\pi$ Solve. Volume of cone B $= 324\pi$

▶ The surface area of cone B is 216π square feet. The volume of cone B is 324π cubic feet.

EXAMPLE 4 Finding the Volume of a Composite Solid

Find the volume of the composite solid.

Let h_1 be the height of the cylinder and let h_2 be the height of the cone.

Volume of solid = Volume of cylinder + Volume of cone

$= \pi r^2 h_1 + \frac{1}{3}\pi r^2 h_2$ Write formulas.

$= \pi \cdot 6^2 \cdot 5 + \frac{1}{3}\pi \cdot 6^2 \cdot 4$ Substitute.

$= 180\pi + 48\pi$ Simplify.

$= 228\pi$ Add.

$= 716.28$ Use technology.

▶ The volume is about 716.28 cubic meters.

In-Class Practice

Self-Assessment

4. Cone C and cone D are similar. Find the surface area and volume of cone D.

5. Find the volume of the composite solid.

1 I don't understand yet. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone.

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Quick Check Exercises 4 and 5

Paul's Notes

Using Similar Solids and Composite Solids

? Pose Example 3 and ask, "How are the surface areas of the similar cones related?" The ratio of their surface areas is 9 : 1. "How are the volumes of the similar cones related?" The ratio of their volumes is 27 : 1.

? **Turn and Talk:** "How can you find the volume of the composite solid in Example 4?"

Talk About It

◎ "Reflect on the last two success criteria. Where are you in your learning?"

Paul's Notes

Closure

- **Exit Ticket:** "Find the surface area and volume of a right cone that has a diameter of 8 centimeters and a height of 3 centimeters." surface area: 36π, or about 113.10 cm^2; volume: 16π, or about 50.27 cm^3
- See *Support for All Learners* for content support to nudge students toward mastery.

EL English Learner Support

Constructing Mathematical Explanations

Review the vocabulary terms from this section and previous sections in this chapter by having students work with a partner to create a definition and example chart. This chart can be used to organize information about a concept. Students fill in the top rectangle with a term and its definition or description. Then students draw and fill in rectangles below the top rectangle with examples to illustrate the term. Students may also use a rectangle to add notes in their home language. Definition and example charts are useful for concepts that can be illustrated with more than one type of example. Support students in the following ways.

Entering–Emerging: Have partners complete a definition and example chart for one vocabulary term and then compare charts with another pair. Students may share ideas and add to their own chart as needed. Ensure students used a labeled image in one of the rectangles. Repeat the steps for each vocabulary term.

Developing–Expanding: Have partners complete a definition and example chart for one vocabulary term and then compare charts with another pair. Students may share ideas and add to their own chart as needed. Repeat the steps for each vocabulary term.

Bridging–Reaching: Have students work with students of varying English language proficiencies to encourage peer collaboration.

My Thoughts on the Lesson

Go online to reflect on this lesson. How did you support student learning? What worked? What didn't work? What would you do differently?

12.4 Support for All Learners

Supporting Student Learning

Use the **Quick Check** exercises to assess understanding of lesson concepts and support emerging learners with additional resources. Remember to check students' Self-Assessment to help inform your instructional decisions.

Reinforce (TIER 1)

Identify where students need support with *this lesson*.

- **Exercise 1:** Find surface areas of cones.
- **Exercise 3:** Find volumes of cones.
- **Exercise 4:** Find the volumes of similar cones.
- **Exercise 5:** Find the volumes of composite solids containing cones.

Re-engage (TIER 2)

Identify where students need support with *prior skills*.

- Find volumes of cylinders. **Geometry Section 12.2**

Intervene (TIER 3)

Identify where students have additional *unfinished learning*.

- Use the formulas for the circumference and area of a circle to find measures. **Geometry Sections 11.1 and 11.2**

Deepening Student Learning

Support proficient and advanced learners with suggested resources targeted to promote more complex thinking and deeper understanding of lesson concepts.

Differentiated Resources

Learning is individualized and students may move in and out of proficiency levels with each skill and concept. The online resources provide a wide range of layered support for the diverse needs in your classroom.

RESOURCES

India's Notes

Equity in Action

Your brightest students may stand out as academic outliers among their peers. These students tend to get bored with the lesson and may not complete their regular assignments. Provide enrichment opportunities and open-ended tasks to allow these students to dive deeper into a topic while keeping them engaged in your class.

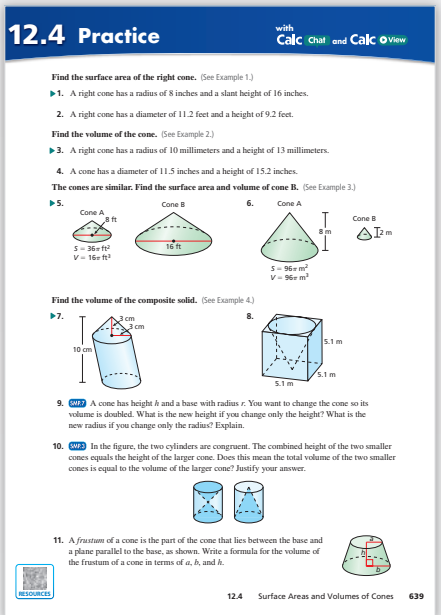
Listen to India talk about her insights and inspiration on equity in action.

12.4 Practice

See the *Answer Guide* for all Practice notes and answers to help inform next steps for your students.

Data Talk

See the *Answer Guide* for suggestions on how to connect the data display to math and facilitate classroom discussion.



12.4 Practice

with Calc Chat and Calc View

Find the surface area of the right cone. (See Example 1.)

1. A right cone has a radius of 8 inches and a slant height of 16 inches.
2. A right cone has a diameter of 11.2 feet and a height of 9.2 feet.

Find the volume of the cone. (See Example 2.)

3. A right cone has a radius of 10 millimeters and a height of 13 millimeters.
4. A cone has a diameter of 11.5 inches and a height of 15.2 inches.

The cones are similar. Find the surface area and volume of cone B. (See Example 3.)

5. Cone A: 8 ft; Cone B: 16 ft; $S = 36\pi$ ft^2, $V = 16\pi$ ft^3
6. Cone A: 8 m; Cone B: 2 m; $S = 96\pi$ m^2, $V = 96\pi$ m^3

Find the volume of the composite solid. (See Example 4.)

7. 3 cm, 3 cm, 10 cm
8. 5.1 m, 5.1 m, 5.1 m

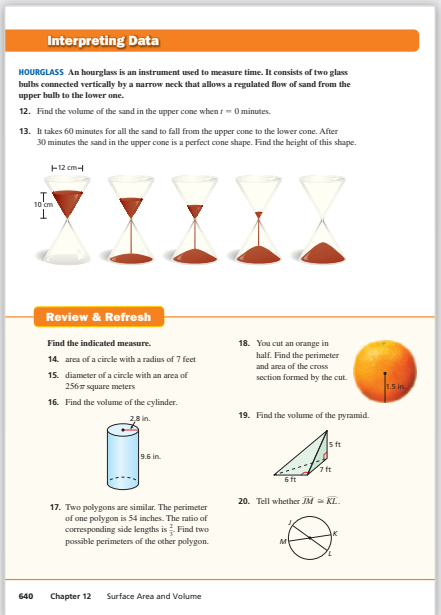
9. SMP 3 A cone has height h and a base with radius r. You want to change the cone so its volume is doubled. What is the new height if you change only the height? What is the new radius if you change only the radius? Explain.

10. SMP 3 In the figure, the two cylinders are congruent. The combined height of the two smaller cones equals the height of the larger cone. Does this mean the total volume of the two smaller cones is equal to the volume of the larger cone? Justify your answer.

11. A *frustum* of a cone is the part of the cone that lies between the base and a plane parallel to the base, as shown. Write a formula for the volume of the frustum of a cone in terms of a, b, and h.

RESOURCES

12.4 Surface Areas and Volumes of Cones 639



Interpreting Data

HOURGLASS An hourglass is an instrument used to measure time. It consists of two glass bulbs connected vertically by a narrow neck that allows a regulated flow of sand from the upper bulb to the lower one.

12. Find the volume of the sand in the upper cone when $t = 0$ minutes.
13. It takes 60 minutes for all the sand to fall from the upper cone to the lower cone. After 30 minutes the sand in the upper cone is a perfect cone shape. Find the height of this shape.

12 cm, 10 cm

Review & Refresh

Find the indicated measure.

14. area of a circle with a radius of 7 feet
15. diameter of a circle with an area of 256π square meters
16. Find the volume of the cylinder. 2.8 in., 9.6 in.
17. Two polygons are similar. The perimeter of one polygon is 54 inches. The ratio of corresponding side lengths is $\frac{2}{3}$. Find two possible perimeters of the other polygon.
18. You cut an orange in half. Find the perimeter and area of the cross section formed by the cut. 1.5 in.
19. Find the volume of the pyramid. 5 ft, 7 ft, 6 ft
20. Tell whether $\overline{JM} \cong \overline{KL}$. J, K, M, L

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Student Edition pages 639 and 640

12.5 Surface Areas and Volumes of Spheres

Learning Target:
Find and use surface areas and volumes of spheres.

Success Criteria:
- Find surface areas of spheres.
- Find volumes of spheres.
- Find the volumes of composite solids.

Prerequisite Skills
- Find volumes of cylinders and cones. **Geometry Sections 12.2 and 12.4**
- Use the formulas for the circumference and area of a circle to find measures. **Geometry Sections 11.1 and 11.2**

Content Standards for Mathematics
Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.

WIDA English Language Development Standards
ELD-MA.9-12.Explain.Expressive Construct mathematical explanations that share solutions with others

ELD-MA.9-12.Argue.Expressive Construct mathematics arguments that generalize logical relationships across cases*

*See the *Answer Guide*.

Paul's Notes

Launch
Goal: Students will estimate the amount of wrapping paper needed to cover real-life spherical objects.

- Bring several spherical objects (e.g., globe, basketball) to class.
- ? "I want to cover these objects with wrapping paper, so I can give them as gifts. How can I determine how much wrapping paper I will need to cover each object?"
- Allow time for students to handle the objects. You could divide the class into small groups, give each group one of the objects, and ask them to make a conjecture. Discuss students' conjectures as a class. Do any students mention that it would be difficult to wrap a spherical object without overlapping the paper?

Talk About It
"To figure out how much wrapping paper is needed to cover a spherical object, you need to know the surface area of the object. In this section, you will learn about a formula that can be used to find the surface area of a sphere."

Use **Quick Check** exercises to quickly identify where students are in their learning.

Paul's Notes

Investigate

Goal: Students will explore the surface area of a sphere and derive a formula for the volume of a sphere.

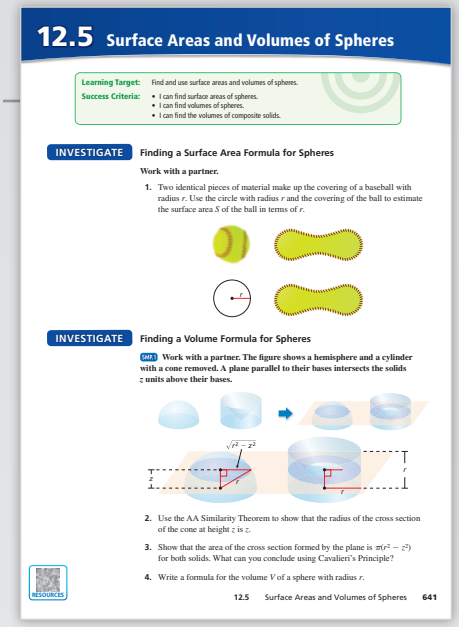
- **Exercise 1:** It would be wonderful if you could provide students with old baseballs or softballs to use as manipulatives. Consider asking a baseball or softball coach if you could borrow some.
- **Exercise 2:** Students should feel comfortable using their understanding of the AA Similarity Theorem. As you circulate, do not give in to the urge to rescue pairs who are struggling. Allow productive struggle to occur and encourage students to persevere.
- If students cannot write a formula for the volume of a sphere, do not despair. The lesson will provide more opportunities for students to develop understanding.

Talk About It

"In the Investigates, you explored the surface area of a sphere and wrote a formula for the volume of a sphere. In the lesson, you will use surface area and volume formulas to solve problems involving spheres."

Differentiating Instruction

- **Emerging:** This is the last section that introduces new surface area and volume formulas. The many formulas in this chapter can be overwhelming for students, so encourage them to create a summary that includes each solid and its surface area and volume formulas. You can increase students' confidence in using the different formulas by asking them to find the surface area and volume of each type of solid. This will help students identify any formulas they are struggling to understand.
- **Proficient:** If students are proficient in using most or all of the surface area and volume formulas, they should not have difficulty with the formulas for a sphere. Have them complete the In-Class Practice exercises with a partner.



12.5 Surface Areas and Volumes of Spheres

Learning Target: Find and use surface areas and volumes of spheres.

Success Criteria:
- I can find surface areas of spheres.
- I can find volumes of spheres.
- I can find the volumes of composite solids.

INVESTIGATE Finding a Surface Area Formula for Spheres

Work with a partner.

1. Two identical pieces of material make up the covering of a baseball with radius r. Use the circle with radius r and the covering of the ball to estimate the surface area S of the ball in terms of r.

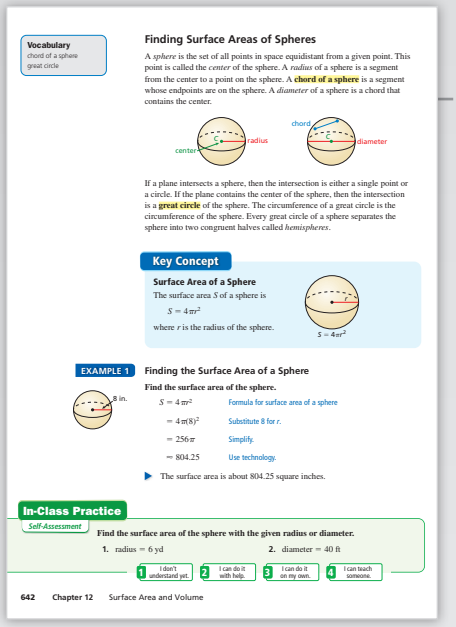
INVESTIGATE Finding a Volume Formula for Spheres

DATA Work with a partner. The figure shows a hemisphere and a cylinder with a cone removed. A plane parallel to their bases intersects the solids z units above their bases.

2. Use the AA Similarity Theorem to show that the radius of the cross section of the cone at height z is z.
3. Show that the area of the cross section formed by the plane is $\pi(r^2 - z^2)$ for both solids. What can you conclude using Cavalieri's Principle?
4. Write a formula for the volume V of a sphere with radius r.

12.5 Surface Areas and Volumes of Spheres 641

Student Edition page 641



Vocabulary
chord of a sphere
great circle

Finding Surface Areas of Spheres

A *sphere* is the set of all points in space equidistant from a given point. This point is called the *center* of the sphere. A *radius* of a sphere is a segment from the center to a point on the sphere. A **chord of a sphere** is a segment whose endpoints are on the sphere. A *diameter* of a sphere is a chord that contains the center.

center C, radius, chord, diameter

If a plane intersects a sphere, then the intersection is either a single point or a circle. If the plane contains the center of the sphere, then the intersection is a **great circle** of the sphere. The circumference of a great circle is the circumference of the sphere. Every great circle of a sphere separates the sphere into two congruent halves called *hemispheres*.

Key Concept

Surface Area of a Sphere
The surface area S of a sphere is
$S = 4\pi r^2$
where r is the radius of the sphere.

$S = 4\pi r^2$

EXAMPLE 1 **Finding the Surface Area of a Sphere**

Find the surface area of the sphere.

8 in.

$S = 4\pi r^2$ Formula for surface area of a sphere
$= 4\pi(8)^2$ Substitute 8 for r.
$= 256\pi$ Simplify.
≈ 804.25 Use technology.

▶ The surface area is about 804.25 square inches.

In-Class Practice
Self-Assessment

Find the surface area of the sphere with the given radius or diameter.

1. radius = 6 yd
2. diameter = 40 ft

1 I don't understand yet. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone.

642 Chapter 12 Surface Area and Volume

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Quick Check Exercise 1

Paul's Notes

Finding Surface Areas of Spheres

- Use one of the objects from the Launch to explain the vocabulary associated with a sphere and the formula in the Key Concept.
- A globe can help students visualize a great circle and two hemispheres. You can also reference air travel and explain that a path along a great circle of Earth represents the shortest path between two locations on Earth's surface.
- Have students work independently to solve Example 1 and Exercises 1 and 2.

EL English Learner Support

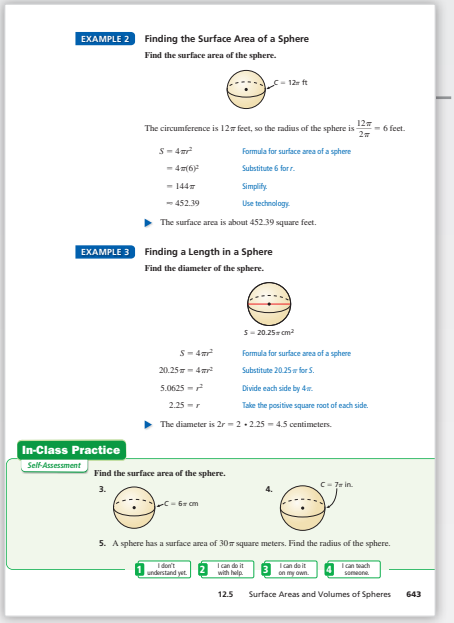
Constructing Mathematical Explanations

After demonstrating Example 1, have students work in pairs as they complete Exercises 1 and 2. Have one partner ask the other, "What is the value of r? What is the surface area of the sphere?" Have partners alternate asking and answering.

Entering–Emerging: As partners complete the exercises, provide sentence frames for students to use to share their solutions with another pair. Students may use short phrases to share their surface areas. "Do your surface areas match? If not, compare your solution steps to find the error."

Developing–Expanding: As partners complete the exercises, provide a word bank for students to use to share their solutions with another pair. Students may use short sentences to share their surface areas. "Do your surface areas match? If not, compare your solution steps to find the error."

Bridging–Reaching: After students complete the exercises, have them share their surface areas with a partner. Encourage students to use complete sentences while comparing solution steps.



EXAMPLE 2 **Finding the Surface Area of a Sphere**

Find the surface area of the sphere.

$C = 12\pi$ ft

The circumference is 12π feet, so the radius of the sphere is $\frac{12\pi}{2\pi} = 6$ feet.

$S = 4\pi r^2$ Formula for surface area of a sphere
$= 4\pi(6)^2$ Substitute 6 for r.
$= 144\pi$ Simplify.
≈ 452.39 Use technology.

▶ The surface area is about 452.39 square feet.

EXAMPLE 3 **Finding a Length in a Sphere**

Find the diameter of the sphere.

$S = 20.25\pi$ cm²

$S = 4\pi r^2$ Formula for surface area of a sphere
$20.25\pi = 4\pi r^2$ Substitute 20.25π for S.
$5.0625 = r^2$ Divide each side by 4π.
$2.25 = r$ Take the positive square root of each side.

▶ The diameter is $2r = 2 \cdot 2.25 = 4.5$ centimeters.

In-Class Practice
Self-Assessment

Find the surface area of the sphere.

3. $C = 6\pi$ cm
4. $C = 7\pi$ in.
5. A sphere has a surface area of 30π square meters. Find the radius of the sphere.

1 I don't understand yet. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone.

12.5 Surface Areas and Volumes of Spheres 643

Paul's Notes

Finding Surface Areas of Spheres

? "If you know the circumference of a great circle of a sphere, can you find the surface area of the sphere? Explain." Yes, you can use the formula for circumference to find the radius, and then find the surface area.

- Allow time for students to work through Examples 2 and 3.

Talk About It

"Reflect on the first success criterion. Can you explain how to find the surface area of a sphere?" If students are uncertain, have them complete Exercises 3–5 with a partner and then share their results with the class.

Student Edition page 643

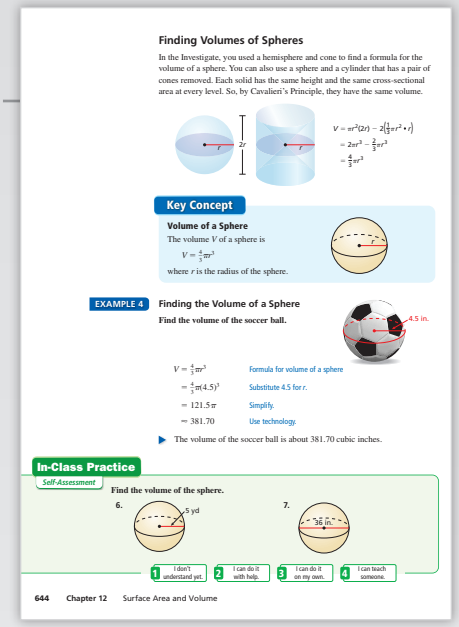
Paul's Notes

Finding Volumes of Spheres

- Remind students of the formula for the volume of a sphere, which should be familiar after completing the second Investigate. Take a moment to discuss why Cavalieri's Principle applies.
- Have students complete Example 4 independently.

Talk About It

"Reflect on the second success criterion. Can you explain how to find the volume of a sphere?" If students are uncertain, have them complete Exercises 6 and 7 with a partner and then share their results with the class.



Finding Volumes of Spheres

In the Investigate, you used a hemisphere and cone to find a formula for the volume of a sphere. You can also use a sphere and a cylinder that has a pair of cones removed. Each solid has the same height and the same cross-sectional area at every level. So, by Cavalieri's Principle, they have the same volume.

$V = \pi r^2(2r) - 2\left(\frac{1}{3}\pi r^2 \cdot r\right)$
$= 2\pi r^3 - \frac{2}{3}\pi r^3$
$= \frac{4}{3}\pi r^3$

Key Concept

Volume of a Sphere

The volume V of a sphere is

$V = \frac{4}{3}\pi r^3$

where r is the radius of the sphere.

EXAMPLE 4 Finding the Volume of a Sphere

Find the volume of the soccer ball.

4.5 in.

$V = \frac{4}{3}\pi r^3$ Formula for volume of a sphere
$= \frac{4}{3}\pi(4.5)^3$ Substitute 4.5 for r.
$= 121.5\pi$ Simplify.
≈ 381.70 Use technology.

▶ The volume of the soccer ball is about 381.70 cubic inches.

In-Class Practice

Self-Assessment

Find the volume of the sphere.

6. 5 yd

7. 36 in.

1 I don't understand yet. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone.

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Quick Check Exercise 6

Paul's Notes

Finding Volumes of Spheres

? Pose Example 5 and say, "You are given the surface area of a sphere. How can you use this information to find the volume of the sphere?" Use the surface area to find the radius and then substitute the value of the radius for r in the volume formula.

? "How can you find the volume of the composite solid in Example 6?" Subtract the volume of the hemisphere from the volume of the cylinder.

- **Four Corners:** Designate each of the four corners in your classroom as one of the following: prisms and cylinders, pyramids, cones, and spheres. Give each student an index card with a problem that involves finding surface area or volume. "Solve the problem on your card. Then move to the corner of the room that corresponds to your card."

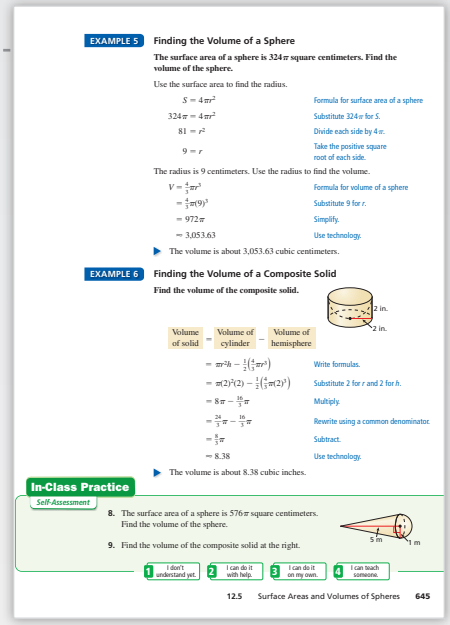
Formative Assessment Tip

Four Corners

This technique allows students to place a problem into one of four categories or answer choices. Designate each of four corners in your classroom as a different category or answer. Give each student an index card with a problem written on it and ask them to move to the corners of the room that correspond to their cards. Have students discuss with others in their corner and determine if they are all in the appropriate corner. Allow students to change corners if necessary. Have each group read their cards to the class, so everyone hears which problems are related to each category or answer. Discuss any discrepancies as a class.

Talk About It

"You have learned to find surface areas and volumes of spheres. You have also found volumes of composite solids that involved spheres. Where are you in your learning? Do you need additional support?"



EXAMPLE 5 Finding the Volume of a Sphere

The surface area of a sphere is 324π square centimeters. Find the volume of the sphere.

Use the surface area to find the radius.

$S = 4\pi r^2$ Formula for surface area of a sphere
$324\pi = 4\pi r^2$ Substitute 324π for S.
$81 = r^2$ Divide each side by 4π.
$9 = r$ Take the positive square root of each side.

The radius is 9 centimeters. Use the radius to find the volume.

$V = \frac{4}{3}\pi r^3$ Formula for volume of a sphere
$= \frac{4}{3}\pi(9)^3$ Substitute 9 for r.
$= 972\pi$ Simplify.
$\approx 3{,}053.63$ Use technology.

▶ The volume is about 3,053.63 cubic centimeters.

EXAMPLE 6 Finding the Volume of a Composite Solid

Find the volume of the composite solid.

2 in.
2 in.

Volume of solid = Volume of cylinder − Volume of hemisphere

$= \pi r^2 h - \frac{1}{2}\left(\frac{4}{3}\pi r^3\right)$ Write formulas.
$= \pi(2)^2(2) - \frac{1}{2}\left(\frac{4}{3}\pi(2)^3\right)$ Substitute 2 for r and 2 for h.
$= 8\pi - \frac{16}{3}\pi$ Multiply.
$= \frac{24}{3}\pi - \frac{16}{3}\pi$ Rewrite using a common denominator.
$= \frac{8}{3}\pi$ Subtract.
≈ 8.38 Use technology.

▶ The volume is about 8.38 cubic inches.

In-Class Practice

Self-Assessment

8. The surface area of a sphere is 576π square centimeters. Find the volume of the sphere.

9. Find the volume of the composite solid at the right.

5 in. 1 in.

1 I don't understand yet. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone.

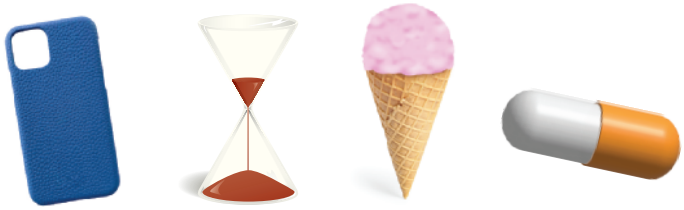
12.5 Surface Areas and Volumes of Spheres 645

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Quick Check Exercise 9

Math Talk

- Display the following real-life objects.



? "Which of these objects can be modeled by a composite solid? How would you find the surface area and volume of each composite solid? What information would you need? Silently and mentally consider the questions. When you are ready to share, show a thumbs up."

- When all students are ready, ask them to share their answers and record all unique answers on the board. Then ask students to share their reasoning so that others can learn from their thought processes.

? Ask probing questions to help students clarify their thinking. For example, "What three-dimensional figures make up the composite solid?"

Paul's Notes

Closure

? **Exit Ticket:** "What are your top two takeaways from this lesson? What two questions are you still struggling to answer?"

- See *Support for All Learners* for content support to nudge students toward mastery.

My Thoughts on the Lesson

Go online to reflect on this lesson. How did you support student learning? What worked? What didn't work? What would you do differently?

12.5 Support for All Learners

Supporting Student Learning

Use the **Quick Check** exercises to assess understanding of lesson concepts and support emerging learners with additional resources. Remember to check students' Self-Assessment to help inform your instructional decisions.

Reinforce (TIER 1)

Identify where students need support with *this lesson*.

- **Exercise 1:** Find surface areas of spheres.
- **Exercise 6:** Find volumes of spheres.
- **Exercise 9:** Find the volumes of composite solids.

Re-engage (TIER 2)

Identify where students need support with *prior skills*.

- Find volumes of cylinders and cones. **Geometry Sections 12.2 and 12.4**

Intervene (TIER 3)

Identify where students have additional *unfinished learning*.

- Use the formulas for the circumference and area of a circle to find measures. **Geometry Sections 11.1 and 11.2**

Deepening Student Learning

Support proficient and advanced learners with suggested resources targeted to promote more complex thinking and deeper understanding of lesson concepts.

Differentiated Resources

Learning is individualized and students may move in and out of proficiency levels with each skill and concept. The online resources provide a wide range of layered support for the diverse needs in your classroom.

RESOURCES

India's Notes

Equity in Action

Some students may become jealous when their peers perform well in class. They might sulk, become quiet, or criticize their peers instead of celebrating them. Help your students recognize their own strengths and set goals by asking, "In what areas do you feel successful? unsuccessful?" Establishing a culture in your classroom that celebrates all students' successes will teach them that their success is independent of another's success.

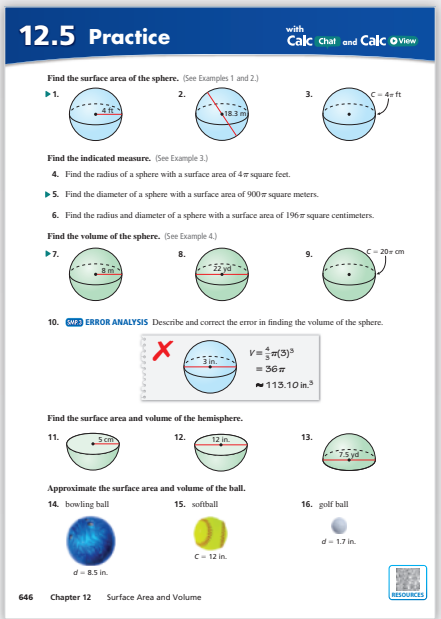
Listen to India talk about her insights and inspiration on equity in action.

12.5 Practice

See the *Answer Guide* for all Practice notes and answers to help inform next steps for your students.

Data Talk

See the *Answer Guide* for suggestions on how to connect the data display to math and facilitate classroom discussion.


12.5 Practice

with Calc Chat and Calc View

Find the surface area of the sphere. (See Examples 1 and 2.)

1. 4 ft
2. 18.3 m
3. $C = 4\pi$ ft

Find the indicated measure. (See Example 3.)

4. Find the radius of a sphere with a surface area of 4π square feet.
5. Find the diameter of a sphere with a surface area of 900π square meters.
6. Find the radius and diameter of a sphere with a surface area of 196π square centimeters.

Find the volume of the sphere. (See Example 4.)

7. 8 in.
8. 22 yd
9. $C = 20\pi$ cm

10. WP3 ERROR ANALYSIS Describe and correct the error in finding the volume of the sphere.

Find the surface area and volume of the hemisphere.

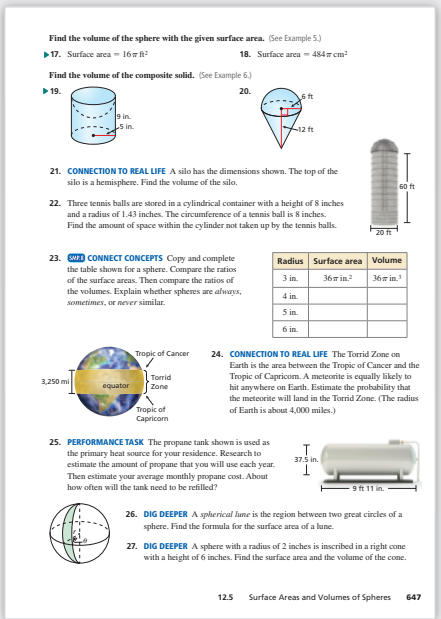
11. 5 cm
12. 12 in.
13. 7.5 yd

Approximate the surface area and volume of the ball.

14. bowling ball, $d = 8.5$ in.
15. softball, $C = 12$ in.
16. golf ball, $d = 1.7$ in.

646 Chapter 12 Surface Area and Volume

RESOURCES


Find the volume of the sphere with the given surface area. (See Example 5.)

17. Surface area $= 16\pi$ ft^2
18. Surface area $= 484\pi$ cm^2

Find the volume of the composite solid. (See Example 6.)

19. 9 in., 5 in.
20. 6 ft, 12 ft

21. CONNECTION TO REAL LIFE A silo has the dimensions shown. The top of the silo is a hemisphere. Find the volume of the silo.

22. Three tennis balls are stored in a cylindrical container with a height of 8 inches and a radius of 1.43 inches. The circumference of a tennis ball is 8 inches. Find the amount of space within the cylinder not taken up by the tennis balls.

60 ft, 20 ft

23. WP3 CONNECT CONCEPTS Copy and complete the table shown for a sphere. Compare the ratios of the surface areas. Then compare the ratios of the volumes. Explain whether spheres are *always*, *sometimes*, or *never* similar.

Radius	Surface area	Volume
3 in.	36π in.^{2}	36π in.^{3}
4 in.		
5 in.		
6 in.		

Tropic of Cancer, Torrid Zone, equator, Tropic of Capricorn, 3,250 mi

24. CONNECTION TO REAL LIFE The Torrid Zone on Earth is the area between the Tropic of Cancer and the Tropic of Capricorn. A meteorite is equally likely to hit anywhere on Earth. Estimate the probability that the meteorite will land in the Torrid Zone. (The radius of Earth is about 4,000 miles.)

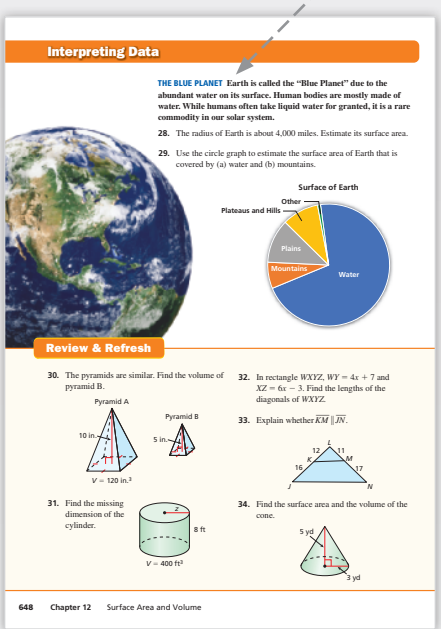
25. PERFORMANCE TASK The propane tank shown is used as the primary heat source for your residence. Research to estimate the amount of propane that you will use each year. Then estimate your average monthly propane cost. About how often will the tank need to be refilled?

37.5 in., 9 ft 11 in.

26. DIG DEEPER A *spherical lune* is the region between two great circles of a sphere. Find the formula for the surface area of a lune.

27. DIG DEEPER A sphere with a radius of 2 inches is inscribed in a right cone with a height of 6 inches. Find the surface area and the volume of the cone.

12.5 Surface Areas and Volumes of Spheres 647


Interpreting Data

THE BLUE PLANET Earth is called the "Blue Planet" due to the abundant water on its surface. Human bodies are mostly made of water. While humans often take liquid water for granted, it is a rare commodity in our solar system.

28. The radius of Earth is about 4,000 miles. Estimate its surface area.

29. Use the circle graph to estimate the surface area of Earth that is covered by (a) water and (b) mountains.

Surface of Earth: Other, Plateaus and Hills, Plains, Mountains, Water

Review & Refresh

30. The pyramids are similar. Find the volume of pyramid B.

Pyramid A: 10 in., $V = 120$ in.^{3}; Pyramid B: 5 in.

31. Find the missing dimension of the cylinder.

8 ft, $V = 400$ ft^3

32. In rectangle $WXYZ$, $WY = 4x + 7$ and $XZ = 6x - 3$. Find the lengths of the diagonals of $WXYZ$.

33. Explain whether $\overline{KM} \parallel \overline{JN}$.

34. Find the surface area and the volume of the cone.

5 yd, 3 yd

648 Chapter 12 Surface Area and Volume

Student Edition pages 646–648

12.6 Modeling with Surface Area and Volume

Learning Target:

Understand the concept of density and modeling with volume.

Success Criteria:

- Explain what density means.
- Use the formula for density to solve problems.
- Use geometric shapes to model objects.
- Solve modeling problems.

Prerequisite Skills

- Find volumes of prisms, cylinders, pyramids, cones, and spheres. **Geometry Sections 12.2–12.5**
- Use two-dimensional figures to model and solve real-life problems. **Geometry Section 11.4**
- Use area formulas to find population densities and solve density problems. **Geometry Section 11.4**
- Solve mathematical and real-life problems involving surface areas of prisms and cylinders. **Grade 7 Section 10.1 and 10.2**

Content Standards for Mathematics

- Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).
- Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).

WIDA English Language Development Standards

ELD-MA.9-12.Explain.Expressive Construct mathematical explanations that describe data and/or an approach used to solve a problem

ELD-MA.9-12.Explain.Expressive Construct mathematical explanations that share solutions with others

Paul's Notes

Launch

Goal: Students will compare densities of real-life objects.

- I like to begin this section with a quick demonstration. Borrow a one-liter beaker from a science colleague. Find pairs of objects in your classroom that have similar volumes but different densities. If you do not have many objects, a science colleague can probably help you.
- Fill the beaker with water. Drop two objects with similar volumes (e.g., a small tea light and a stack of 4 quarters) into the water.
- Students should be able to explain that when the object has a density greater than about 1 gram per cubic centimeter (the density of fresh water) the object will sink, otherwise the object will float.

Talk About It

- Discuss how density is calculated, including the fact that when two of the three variables are known, you can solve for the unknown variable.

Use **Quick Check** exercises to quickly identify where students are in their learning.

Paul's Notes

Investigate

Goal: Students will find densities of real-life objects.

- Students are well-versed in finding volumes of various solids, so they should move through Exercises 1 and 2 rather quickly.
- The volume calculation for the cork will be difficult for students. They need to recognize that similar solids are involved. A part of a cone that contains the vertex has been removed, leaving a solid known as a *frustrum* of a cone. Because the diameter of the base of the part that was removed is one-half the diameter of the original cone, the scale factor is $\frac{1}{2}$ and the volumes will be in the ratio of $\frac{1}{8}$. With $\frac{1}{8}$ of the cone removed, the volume of the frustrum is $\frac{7}{8}$ of the volume of the original cone.

? SMP.6 **Think-Pair-Share:** "Explain what the units represent for one of the densities you calculated in Exercise 1. Are any of these densities better represented using different units? Explain."

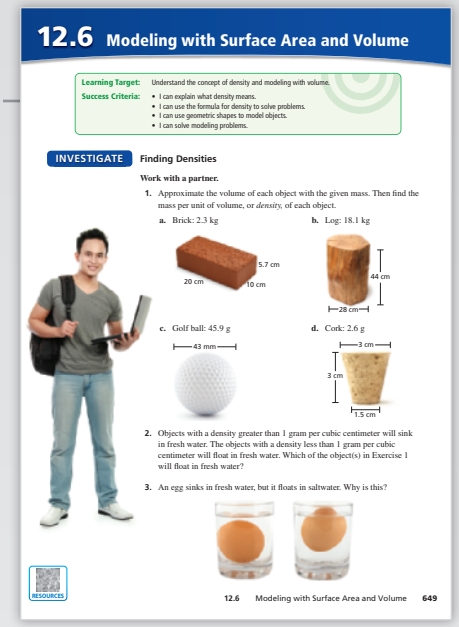
? After Exercise 3, ask, "Have you ever swam in fresh water? saltwater? If you have swum in both, did you notice a difference? Is there a difference in the densities of the different types of water?" Students should recognize that this is why the egg sinks in fresh water, but it floats in saltwater.

Talk About It

"In the lesson, you will use your understanding of volume and surface area formulas to solve real-life problems, some of which are related to density. You will also use your algebraic skills to solve equations."

Differentiating Instruction

- **Emerging:** Struggling students often forget to include units when they apply surface area and volume formulas to solve modeling problems. Make sure students pay attention to the units given in a problem and determine whether they must convert from one unit to another.
- **Proficient:** Students can use their algebraic skills to work with the formulas in this lesson. Have students complete the In-Class Practice exercises with a partner.



12.6 Modeling with Surface Area and Volume

Learning Target: Understand the concept of density and modeling with volume.

Success Criteria:
- I can explain what density means.
- I can use the formula for density to solve problems.
- I can use geometric shapes to model objects.
- I can solve modeling problems.

INVESTIGATE Finding Densities

Work with a partner.

1. Approximate the volume of each object with the given mass. Then find the mass per unit of volume, or *density*, of each object.

a. Brick: 2.3 kg
20 cm, 10 cm, 5.7 cm

b. Log: 18.1 kg
44 cm, 28 cm

c. Golf ball: 45.9 g
43 mm

d. Cork: 2.6 g
3 cm, 3 cm, 1.5 cm

2. Objects with a density greater than 1 gram per cubic centimeter will sink in fresh water. The objects with a density less than 1 gram per cubic centimeter will float in fresh water. Which of the object(s) in Exercise 1 will float in fresh water?

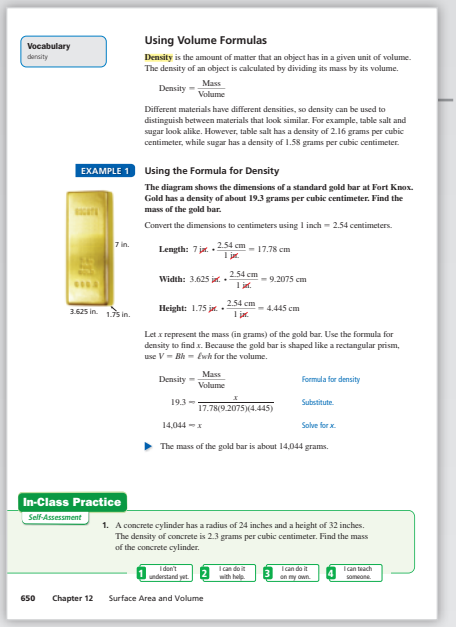
3. An egg sinks in fresh water, but it floats in saltwater. Why is this?

RESOURCES

12.6 Modeling with Surface Area and Volume 649

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Quick Check Investigate Exercise 3



Vocabulary
density

Using Volume Formulas

Density is the amount of matter that an object has in a given unit of volume. The density of an object is calculated by dividing its mass by its volume.

$$\text{Density} = \frac{\text{Mass}}{\text{Volume}}$$

Different materials have different densities, so density can be used to distinguish between materials that look similar. For example, table salt and sugar look alike. However, table salt has a density of 2.16 grams per cubic centimeter, while sugar has a density of 1.58 grams per cubic centimeter.

EXAMPLE 1 **Using the Formula for Density**

The diagram shows the dimensions of a standard gold bar at Fort Knox. Gold has a density of about 19.3 grams per cubic centimeter. Find the mass of the gold bar.

7 in.
3.625 in.
1.75 in.

Convert the dimensions to centimeters using 1 inch = 2.54 centimeters.

Length: $7 \text{ in.} \cdot \frac{2.54 \text{ cm}}{1 \text{ in.}} = 17.78 \text{ cm}$

Width: $3.625 \text{ in.} \cdot \frac{2.54 \text{ cm}}{1 \text{ in.}} = 9.2075 \text{ cm}$

Height: $1.75 \text{ in.} \cdot \frac{2.54 \text{ cm}}{1 \text{ in.}} = 4.445 \text{ cm}$

Let x represent the mass (in grams) of the gold bar. Use the formula for density to find x. Because the gold bar is shaped like a rectangular prism, use $V = Bh = \ell wh$ for the volume.

$$\text{Density} = \frac{\text{Mass}}{\text{Volume}} \qquad \text{Formula for density}$$

$$19.3 \approx \frac{x}{17.78(9.2075)(4.445)} \qquad \text{Substitute.}$$

$$14{,}044 \approx x \qquad \text{Solve for } x.$$

▶ The mass of the gold bar is about 14,044 grams.

In-Class Practice
Self-Assessment

1. A concrete cylinder has a radius of 24 inches and a height of 32 inches. The density of concrete is 2.3 grams per cubic centimeter. Find the mass of the concrete cylinder.

1 I don't understand yet. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone.

650 Chapter 12 Surface Area and Volume

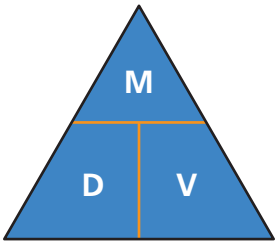
Student Edition page 650

Quick Check Exercise 1

Paul's Notes

Using Volume Formulas

- Define *density* and display the density triangle, which is a helpful tool for working with the density formula. When you cover up the variable you are looking for, the calculation for finding the value of the variable is revealed.



- Have students read the problem in Example 1.

? "In Example 1, the density is given in grams per cubic centimeter, but the dimensions are provided in inches. What must you do to find the mass of the gold bar?" Students should note that they need to convert the dimensions from inches to centimeters before they can find the volume of the gold bar and then its mass.

? **Extension:** "If gold is worth about $1,925 per ounce, what is the value of a standard gold bar at Fort Knox?" about $946,214.50, or about $955,289.75

EL English Learner Support

Constructing Mathematical Explanations

Review the meaning of the word *density*, which students learned in the previous chapter (population density). Display images and explain that *density* is also used in science to describe the amount of matter that an object has in a given unit of volume. Support students with Exercise 1 in the following ways.

Entering–Emerging: Help students draw a diagram and write an equation to find the mass of the cylinder. Provide a sentence frame for students to use to write their answer.

Developing–Expanding: Have partners draw a diagram and write an equation to find the mass of the cylinder. Provide a sentence frame for students to use to write their answer and then have partners share their answer with another pair.

Bridging–Reaching: Have students find the mass of the cylinder and write their answer using a complete sentence. Then have students share their answer with a partner.

Paul's Notes

Using Volume Formulas

- Allow time for students to read each part of Example 2. It is reasonable for students to complete part (a) independently, but part (b) is much more involved.

? Before students begin part (b), ask, "How do the units in the first question provide a clue about solving this problem?"

? **Turn and Talk:** "What information would be helpful to record in a table?" Allow students to work with their partner to make a table. They should notice linear trends in the rows for the height and base radius but not in the row for volume.

? "What do you notice about the differences between the volumes from year to year?" They are increasing, so growth in the amount of wood is not linear.

Data Talk

? "What do you notice about the data in Example 2? What do you wonder?"

? "Why might scientists want to know this information?" *Sample answer:* If they know how fast trees are growing, they can make predictions about the future of the forest.

India's Notes

Talk About Careers

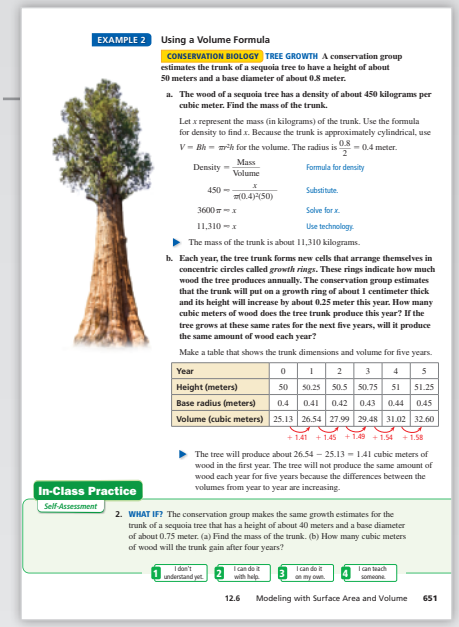
"Conservation biologists usually obtain their bachelor's degree in biology, ecology, environmental science, or a related field. Conservation biologists spend much of their time outdoors collecting data. They may face difficult terrain and extreme temperatures."

? "What other kinds of dangers might conservation biologists face?"

? "What kinds of conservation efforts are ongoing in your area?"

Talk About It

◉ "Do you understand density and how it can be used to solve problems, or do you need to see additional examples?"



EXAMPLE 2 Using a Volume Formula

CONSERVATION BIOLOGY TREE GROWTH A conservation group estimates the trunk of a sequoia tree to have a height of about 50 meters and a base diameter of about 0.8 meter.

a. The wood of a sequoia tree has a density of about 450 kilograms per cubic meter. Find the mass of the trunk.

Let x represent the mass (in kilograms) of the trunk. Use the formula for density to find x. Because the trunk is approximately cylindrical, use $V = Bh = \pi r^2 h$ for the volume. The radius is $\frac{0.8}{2} = 0.4$ meter.

$\text{Density} = \frac{\text{Mass}}{\text{Volume}}$ Formula for density

$450 = \frac{x}{\pi(0.4)^2(50)}$ Substitute.

$3600\pi = x$ Solve for x.

$11{,}310 \approx x$ Use technology.

▶ The mass of the trunk is about 11,310 kilograms.

b. Each year, the tree trunk forms new cells that arrange themselves in concentric circles called *growth rings*. These rings indicate how much wood the tree produces annually. The conservation group estimates that the trunk will put on a growth ring of about 1 centimeter thick and its height will increase by about 0.25 meter this year. How many cubic meters of wood does the tree trunk produce this year? If the tree grows at these same rates for the next five years, will it produce the same amount of wood each year?

Make a table that shows the trunk dimensions and volume for five years.

Year	0	1	2	3	4	5
Height (meters)	50	50.25	50.5	50.75	51	51.25
Base radius (meters)	0.4	0.41	0.42	0.43	0.44	0.45
Volume (cubic meters)	25.13	26.54	27.99	29.48	31.02	32.60
		+1.41	+1.45	+1.49	+1.54	+1.58

▶ The tree will produce about 26.54 − 25.13 = 1.41 cubic meters of wood in the first year. The tree will not produce the same amount of wood each year for five years because the differences between the volumes from year to year are increasing.

In-Class Practice

Self-Assessment

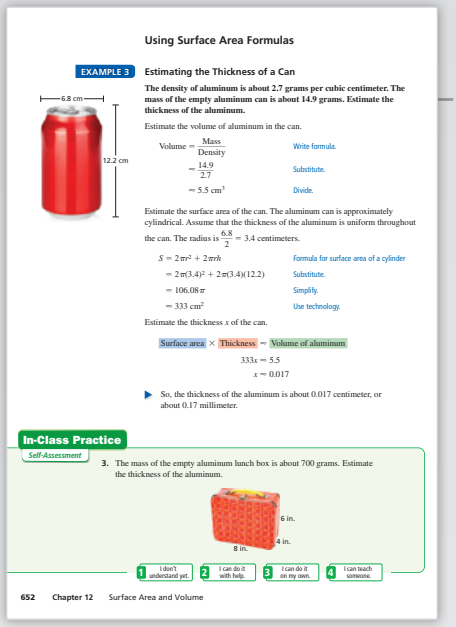
2. WHAT IF? The conservation group makes the same growth estimates for the trunk of a sequoia tree that has a height of about 40 meters and a base diameter of about 0.75 meter. (a) Find the mass of the trunk. (b) How many cubic meters of wood will the trunk gain after four years?

1 I don't understand yet. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone.

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Quick Check Exercise 2



Using Surface Area Formulas

EXAMPLE 3 Estimating the Thickness of a Can

The density of aluminum is about 2.7 grams per cubic centimeter. The mass of the empty aluminum can is about 14.9 grams. Estimate the thickness of the aluminum.

Estimate the volume of aluminum in the can.

$$\text{Volume} = \frac{\text{Mass}}{\text{Density}} \quad \text{Write formula.}$$
$$= \frac{14.9}{2.7} \quad \text{Substitute.}$$
$$\approx 5.5 \text{ cm}^3 \quad \text{Divide.}$$

Estimate the surface area of the can. The aluminum can is approximately cylindrical. Assume that the thickness of the aluminum is uniform throughout the can. The radius is $\frac{6.8}{2} = 3.4$ centimeters.

$$S = 2\pi r^2 + 2\pi rh \quad \text{Formula for surface area of a cylinder}$$
$$\approx 2\pi(3.4)^2 + 2\pi(3.4)(12.2) \quad \text{Substitute.}$$
$$\approx 106.08\pi \quad \text{Simplify.}$$
$$\approx 333 \text{ cm}^2 \quad \text{Use technology.}$$

Estimate the thickness x of the can.

Surface area × Thickness ≈ Volume of aluminum

$$333x \approx 5.5$$
$$x \approx 0.017$$

So, the thickness of the aluminum is about 0.017 centimeter, or about 0.17 millimeter.

In-Class Practice

Self-Assessment

3. The mass of the empty aluminum lunch box is about 700 grams. Estimate the thickness of the aluminum.

6 in., 4 in., 8 in.

1 I don't understand yet. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone.

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Quick Check Exercise 3

Paul's Notes

Using Surface Area Formulas

? Hold an empty aluminum can and say, "In some states, this can could be recycled for 5 cents. Do you think there is 5 cents worth of aluminum in the can?"

- Read the problem in Example 3 and display the diagram.

? "Will finding the volume of the can help you solve Example 3? Explain." No, you need to find the volume of the aluminum. Allow time for students to work with a partner to solve the problem.

- Have students complete Exercise 3 with a partner. Then discuss their results as a class.

EL English Learner Support

Constructing Mathematical Explanations

Build EL students' background knowledge for Example 3 by showing images of items made from aluminum. Support students with Exercise 3 in the following ways.

Entering–Emerging: Assist students with writing and solving equations to find the thickness of the aluminum. Provide a sentence frame for students to use to write their answer. Have partners read their sentence to another pair.

Developing–Expanding: Have partners write and solve equations. Then have them use the wording from the problem to state their answer using a complete sentence. Have partners read their sentence to another pair and evaluate each other's use of language.

Bridging–Reaching: Have students write their answer using a complete sentence and then read it to a partner. Partners should evaluate each other's use of language.

Talk About It

◎ "How confident are you in using density, surface area, and volume to solve modeling problems?"

Paul's Notes

Closure

- "Spend one minute reviewing your notes from today's lesson." Encourage students to think about parts of the lesson that had positive or negative impacts on them.
- "Spend one minute summarizing as many key details as you can." Allow time for students to share their summaries with the class. Although students cannot capture everything from a lesson in one minute, this sharing opportunity gives students a chance to learn from each other and to add other key details they did not include in their summaries.
- See *Support for All Learners* for content support to nudge students toward mastery.

My Thoughts on the Lesson

Go online to reflect on this lesson. How did you support student learning? What worked? What didn't work? What would you do differently?

12.6 Support for All Learners

Supporting Student Learning

Use the **Quick Check** exercises to assess understanding of lesson concepts and support emerging learners with additional resources. Remember to check students' Self-Assessment to help inform your instructional decisions.

Reinforce (TIER 1)

Identify where students need support with *this lesson*.

- **Investigate Exercise 3:** Explain what density means.
- **Exercise 1:** Use the formula for density to solve problems.
- **Exercise 2:** Use geometric shapes to model objects.
- **Exercise 3:** Solve modeling problems.

Re-engage (TIER 2)

Identify where students need support with *prior skills*.

- Find volumes of prisms, cylinders, pyramids, cones, and spheres. **Geometry Sections 12.2–12.5**

Intervene (TIER 3)

Identify where students have additional *unfinished learning*.

- Use area formulas to solve density problems. **Geometry Section 11.4**

Deepening Student Learning

Support proficient and advanced learners with suggested resources targeted to promote more complex thinking and deeper understanding of lesson concepts.

Differentiated Resources

Learning is individualized and students may move in and out of proficiency levels with each skill and concept. The online resources provide a wide range of layered support for the diverse needs in your classroom.

RESOURCES

India's Notes

Equity in Action

For various reasons, you may have students who are frequently absent from school. These absences can cause students to become despondent and fall behind. Help your students organize a list of incomplete assignments and make a plan to complete the assignments. Then encourage them to dig in and get the work done. When students are caught up in school, they are more likely to stay engaged and attend school.

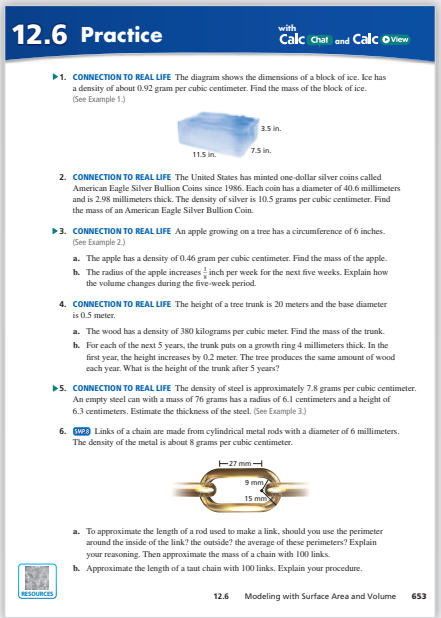
Listen to India talk about her insights and inspiration on equity in action.

12.6 Practice

See the *Answer Guide* for all Practice notes and answers to help inform next steps for your students.

Data Talk

See the *Answer Guide* for suggestions on how to connect the data display to math and facilitate classroom discussion.



12.6 Practice

with Calc Chat and Calc View

▶1. CONNECTION TO REAL LIFE The diagram shows the dimensions of a block of ice. Ice has a density of about 0.92 gram per cubic centimeter. Find the mass of the block of ice. (See Example 1.)

3.5 in., 11.5 in., 7.5 in.

2. CONNECTION TO REAL LIFE The United States has minted one-dollar silver coins called American Eagle Silver Bullion Coins since 1986. Each coin has a diameter of 40.6 millimeters and is 2.98 millimeters thick. The density of silver is 10.5 grams per cubic centimeter. Find the mass of an American Eagle Silver Bullion Coin.

▶3. CONNECTION TO REAL LIFE An apple growing on a tree has a circumference of 6 inches. (See Example 2.)

a. The apple has a density of 0.46 gram per cubic centimeter. Find the mass of the apple.

b. The radius of the apple increases $\frac{1}{8}$ inch per week for the next five weeks. Explain how the volume changes during the five-week period.

4. CONNECTION TO REAL LIFE The height of a tree trunk is 20 meters and the base diameter is 0.5 meter.

a. The wood has a density of 380 kilograms per cubic meter. Find the mass of the trunk.

b. For each of the next 5 years, the trunk puts on a growth ring 4 millimeters thick. In the first year, the height increases by 0.2 meter. The tree produces the same amount of wood each year. What is the height of the trunk after 5 years?

▶5. CONNECTION TO REAL LIFE The density of steel is approximately 7.8 grams per cubic centimeter. An empty steel can with a mass of 76 grams has a radius of 6.1 centimeters and a height of 6.3 centimeters. Estimate the thickness of the steel. (See Example 3.)

6. MP2 Links of a chain are made from cylindrical metal rods with a diameter of 6 millimeters. The density of the metal is about 8 grams per cubic centimeter.

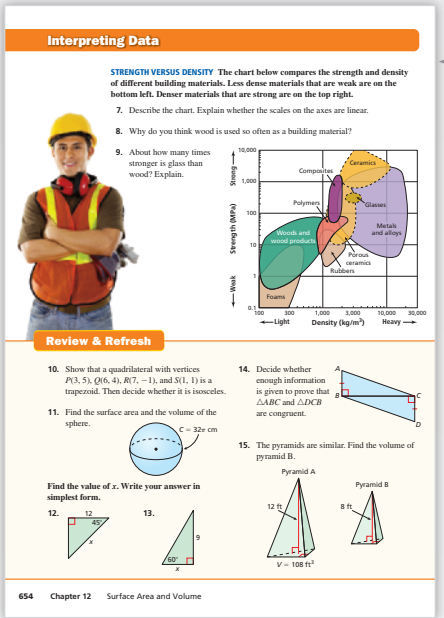
27 mm, 9 mm, 15 mm

a. To approximate the length of a rod used to make a link, should you use the perimeter around the inside of the link? the outside? the average of these perimeters? Explain your reasoning. Then approximate the mass of a chain with 100 links.

b. Approximate the length of a taut chain with 100 links. Explain your procedure.

RESOURCES

12.6 Modeling with Surface Area and Volume 653



Interpreting Data

STRENGTH VERSUS DENSITY The chart below compares the strength and density of different building materials. Less dense materials that are weak are on the bottom left. Denser materials that are strong are on the top right.

7. Describe the chart. Explain whether the scales on the axes are linear.

8. Why do you think wood is used so often as a building material?

9. About how many times stronger is glass than wood? Explain.

Strength (MPa): 0.1, 1, 10, 100, 1,000, 10,000 (Weak → Strong)

Density (kg/m³): 100, 300, 1,000, 3,000, 10,000, 30,000 (Light → Heavy)

Ceramics, Composites, Polymers, Glasses, Metals and alloys, Woods and wood products, Porous ceramics, Rubbers, Foams

Review & Refresh

10. Show that a quadrilateral with vertices $P(3, 5)$, $Q(6, 4)$, $R(7, -1)$, and $S(1, 1)$ is a trapezoid. Then decide whether it is isosceles.

11. Find the surface area and the volume of the sphere. $C = 32\pi$ cm

Find the value of x. Write your answer in simplest form.

12. 12, 45°, x

13. 9, 60°, x

14. Decide whether enough information is given to prove that $\triangle ABC$ and $\triangle DCB$ are congruent. A, B, C, D

15. The pyramids are similar. Find the volume of pyramid B.

Pyramid A: 12 ft, $V = 108$ ft³; Pyramid B: 8 ft

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Student Edition pages 653 and 654

12.7 Solids of Revolution

Learning Target:
Sketch and use solids of revolution.

Success Criteria:
- Sketch and describe solids of revolution.
- Find surface areas and volumes of solids of revolution.
- Form solids of revolution in the coordinate plane.

Prerequisite Skills
- Find volumes and surface areas of cylinders, cones, and spheres. **Geometry Sections 12.2, 12.4, and 12.5**
- Describe and draw cross sections of solids. **Geometry Section 12.1**

Content Standards for Mathematics
Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.

WIDA English Language Development Standards
ELD-MA.9-12.Argue.Expressive Construct mathematics arguments that justify (and refute) conclusions with evidence and mathematical principles

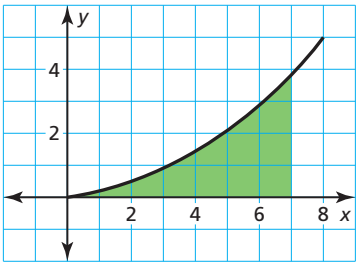
ELD-MA.9-12.Explain.Interpretive Interpret mathematical explanations by identifying a concept or an entity

Paul's Notes

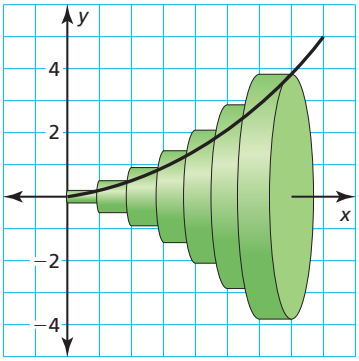
Launch

Goal: Students will sketch and explore a solid of revolution.

- Display the two-dimensional figure shown.

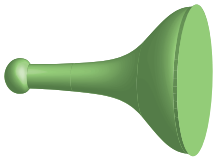


- "Do your best to sketch the solid produced by rotating the figure around the *x*-axis." This might confuse some students, and that is okay. You want to engage and extend students' spatial skills to help prepare them for the lesson.
- After students have completed their sketches, display the solid shown. "The solid you sketched should show smooth curves instead of cylinders, but this composite solid can be used to approximate the solid you drew."



? "How can you find the volume of this composite solid?" Find the sum of the volumes of the cylinders.

- Compare the solid that students drew to a game piece from a popular board game.
- You can spark students' interest in future mathematics by telling them that in a calculus course, they can learn to apply calculus along with volume formulas to find the volume of any solid of revolution!



Talk About It
"In this section, you will learn to sketch and describe solids of revolutions."

Use **Quick Check** exercises to quickly identify where students are in their learning.

Paul's Notes

Investigate

Goal: Students will model solids of revolution.

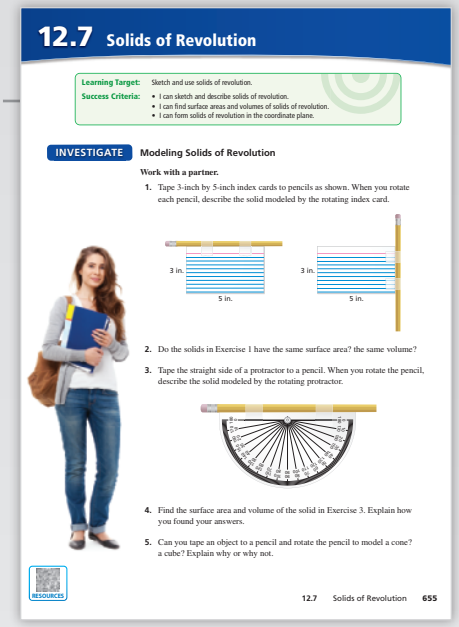
- The physical models in this Investigate will help students develop understanding of the types of solids that can be formed by rotating two-dimensional figures.
- In Exercise 2, students may say that the solids have the same surface area and/or volume. Although each revolution produces a cylinder, the two cylinders have different surface areas and volumes because they have different radii.
- In Exercise 3, students should observe that the revolution produces a sphere. You may need to point out that the pencil and the portion of the protractor below it should not be included in their surface area and volume calculations.
- Solicit multiple responses for Exercise 5. Do students recognize that an object in the shape of a right triangle can be rotated to produce a cone? Do they realize that you cannot rotate a two-dimensional shape around an axis to produce a solid with straight edges?

Talk About It

"In the lesson, you will use your spatial skills to sketch and describe solids of revolution. Then you will use your knowledge of surface area and volume formulas to find surface areas and volumes of solids of revolution."

Differentiating Instruction

- **Emerging:** Students with weak spatial skills often struggle to sketch and describe solids of revolution. The physical models they explored in the Investigate will be a great support. Consider looking for online apps that allow students to explore additional solids of revolutions.
- **Proficient:** Students can use their spatial skills to sketch and describe solids of revolution. They are also proficient with surface area and volume formulas. Have students complete In-Class Practice Exercises 1–4 with a partner and then work through Example 3 as a class.



12.7 Solids of Revolution

Learning Target: Sketch and use solids of revolution.

Success Criteria:
- I can sketch and describe solids of revolution.
- I can find surface areas and volumes of solids of revolution.
- I can form solids of revolution in the coordinate plane.

INVESTIGATE Modeling Solids of Revolution

Work with a partner.

1. Tape 3-inch by 5-inch index cards to pencils as shown. When you rotate each pencil, describe the solid modeled by the rotating index card.

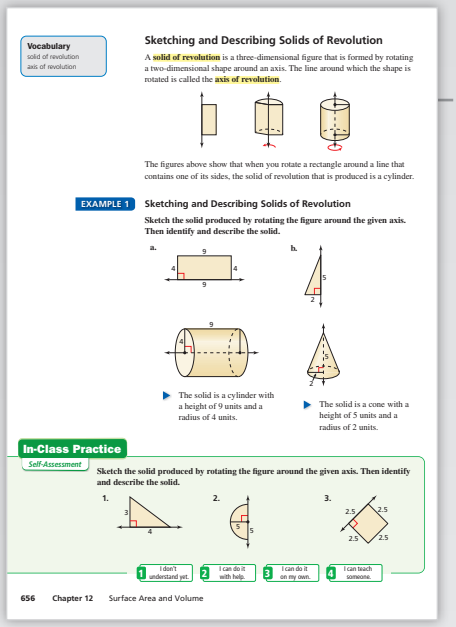
3 in. 5 in. 3 in. 5 in.

2. Do the solids in Exercise 1 have the same surface area? the same volume?
3. Tape the straight side of a protractor to a pencil. When you rotate the pencil, describe the solid modeled by the rotating protractor.
4. Find the surface area and volume of the solid in Exercise 3. Explain how you found your answers.
5. Can you tape an object to a pencil and rotate the pencil to model a cone? a cube? Explain why or why not.

RESOURCES

12.7 Solids of Revolution 655

Student Edition page 655



Vocabulary
solid of revolution
axis of revolution

Sketching and Describing Solids of Revolution

A **solid of revolution** is a three-dimensional figure that is formed by rotating a two-dimensional shape around an axis. The line around which the shape is rotated is called the **axis of revolution**.

The figures above show that when you rotate a rectangle around a line that contains one of its sides, the solid of revolution that is produced is a cylinder.

EXAMPLE 1 **Sketching and Describing Solids of Revolution**

Sketch the solid produced by rotating the figure around the given axis. Then identify and describe the solid.

a. b.

The solid is a cylinder with a height of 9 units and a radius of 4 units.

The solid is a cone with a height of 5 units and a radius of 2 units.

In-Class Practice

Self-Assessment

Sketch the solid produced by rotating the figure around the given axis. Then identify and describe the solid.

1. 2. 3.

1 I don't understand yet. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone.

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Quick Check Exercise 1

Paul's Notes

Sketching and Describing Solids of Revolution

- Introduce the vocabulary terms *solid of revolution* and *axis of revolution*. Make connections between the Investigate and the diagram shown at the top of the page.
- Have students complete Example 1 with a partner.

? "If you rotate the rectangle in part (a) around the side that is 4 units long, will the resulting cylinder be different? Explain." Yes, it will have a different radius and height.

EL English Learner Support

Constructing Mathematical Arguments

As you work through this section, remind EL students to add the vocabulary terms, with images and examples, to their math vocabulary notebooks. Support students with Exercises 1–3 in the following ways.

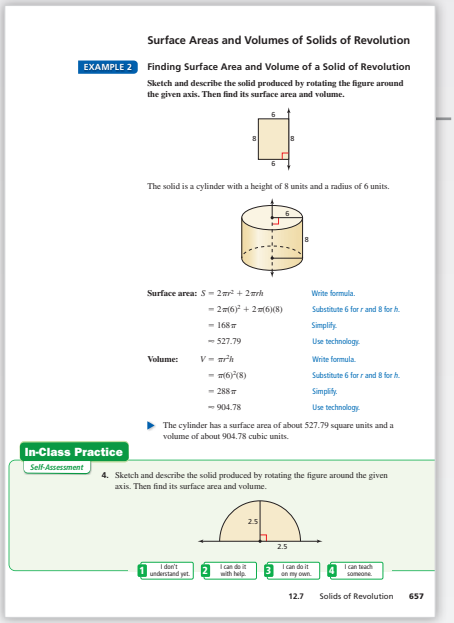
Entering–Emerging: Provide students with sentence frames to guide them with writing their descriptions of the solids. As students are reading a description aloud, have them point to the part of their sketch that they are describing.

Developing–Expanding: Have partners use Example 1 to guide them with writing their descriptions of the solids. As students are reading a description aloud, have them point to the part of their sketch that they are describing.

Bridging–Reaching: Have students write to describe each solid and then share their descriptions with a partner. "Are there details you can add to your descriptions?"

Talk About It

"Are you confident that you can sketch and describe solids of revolutions?" If not, students should use Exercises 1–3 to solidify their understanding.



Surface Areas and Volumes of Solids of Revolution

EXAMPLE 2 **Finding Surface Area and Volume of a Solid of Revolution**

Sketch and describe the solid produced by rotating the figure around the given axis. Then find its surface area and volume.

The solid is a cylinder with a height of 8 units and a radius of 6 units.

Surface area: $S = 2\pi r^2 + 2\pi rh$ Write formula.
$= 2\pi(6)^2 + 2\pi(6)(8)$ Substitute 6 for r and 8 for h.
$= 168\pi$ Simplify.
≈ 527.79 Use technology.

Volume: $V = \pi r^2 h$ Write formula.
$= \pi(6)^2(8)$ Substitute 6 for r and 8 for h.
$= 288\pi$ Simplify.
≈ 904.78 Use technology.

The cylinder has a surface area of about 527.79 square units and a volume of about 904.78 cubic units.

In-Class Practice

Self-Assessment

4. Sketch and describe the solid produced by rotating the figure around the given axis. Then find its surface area and volume.

1 I don't understand yet. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone.

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Quick Check Exercise 4

Paul's Notes

Surface Areas and Volumes of Solids of Revolution

- Have students work with a partner to complete Example 2.
- Make sure students begin by sketching and describing the solid of revolution. They must identify the solid before they can determine which surface area and volume formulas they should use.
- Allow time for each pair to share their work with the class.

Talk About It

Visitor Explanation: "If a visitor entered the room right now, how would you explain how to find the surface area and volume of a solid of revolution?"

Paul's Notes

Forming Solids of Revolution in the Coordinate Plane

- Have students read the problem in Example 3. Then have partners discuss what the enclosed region might look like.
- Emphasize the process for sketching a solid of revolution in the coordinate plane.
  - **Step 1:** Graph each equation and shade the enclosed region.
  - **Step 2:** Reflect the region in the indicated axis.
  - **Step 3:** Connect the vertices of the two regions using curved lines.
- After sketching the solid of revolution, students can identify the solid or composite solid that has been formed and the formula(s) they should use to find its volume. Do students recognize that the solid in Example 3 is a composite solid?

Talk About It

Review the last success criterion with students. "Are you confident in your ability to form solids of revolution in the coordinate plane?"

Paul's Notes

Closure

- **Exit Ticket:** "A triangle with side lengths of 3, 4, and 5 is rotated around one of its legs. Describe the solid." *Sample answer:* a cone with a height of 3 units and a radius of 4 units
- See *Support for All Learners* for content support to nudge students toward mastery.

EL English Learner Support

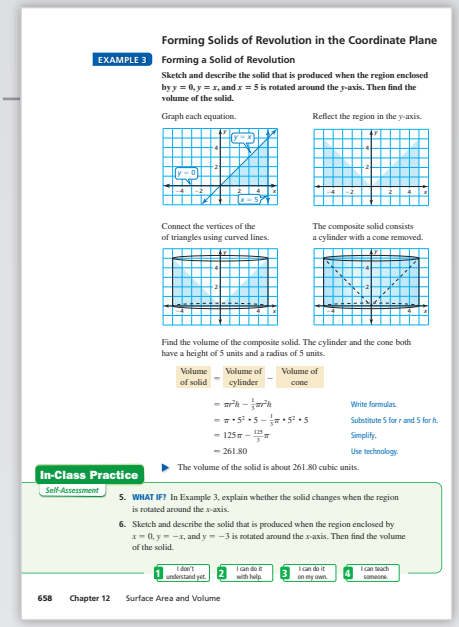
Interpreting Mathematical Explanations

Have EL students review this chapter's math terms by creating a crossword puzzle and a list of clues. Students can use the Internet to search for a puzzle-maker or create the crossword on paper. Then have students trade puzzles to solve. The terms may include new vocabulary, or any other math terms used throughout this chapter.

Emerging: Have students choose 7–10 terms and create clues. Clues may be written descriptions, examples, or images as appropriate.

Expanding: Have students choose 10–15 terms and create clues. Clues may be written descriptions, examples, or images as appropriate.

Bridging: Have students choose 15 terms and create written descriptions or examples as clues.



Forming Solids of Revolution in the Coordinate Plane

EXAMPLE 3 Forming a Solid of Revolution

Sketch and describe the solid that is produced when the region enclosed by $y = 0$, $y = x$, and $x = 5$ is rotated around the y-axis. Then find the volume of the solid.

Graph each equation. Reflect the region in the y-axis.

Connect the vertices of the of triangles using curved lines. The composite solid consists a cylinder with a cone removed.

Find the volume of the composite solid. The cylinder and the cone both have a height of 5 units and a radius of 5 units.

Volume of solid = Volume of cylinder − Volume of cone

$= \pi r^2 h - \frac{1}{3}\pi r^2 h$ Write formulas.

$= \pi \cdot 5^2 \cdot 5 - \frac{1}{3}\pi \cdot 5^2 \cdot 5$ Substitute 5 for r and 5 for h.

$= 125\pi - \frac{125}{3}\pi$ Simplify.

$= 261.80$ Use technology.

▶ The volume of the solid is about 261.80 cubic units.

In-Class Practice

Self-Assessment

5. **WHAT IF?** In Example 3, explain whether the solid changes when the region is rotated around the x-axis.
6. Sketch and describe the solid that is produced when the region enclosed by $x = 0$, $y = -x$, and $y = -3$ is rotated around the x-axis. Then find the volume of the solid.

1 I don't understand yet. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone.

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Quick Check Exercise 6

My Thoughts on the Lesson

Go online to reflect on this lesson. How did you support student learning? What worked? What didn't work? What would you do differently?

12.7 Support for All Learners

Supporting Student Learning

Use the **Quick Check** exercises to assess understanding of lesson concepts and support emerging learners with additional resources. Remember to check students' Self-Assessment to help inform your instructional decisions.

Reinforce (TIER 1)

Identify where students need support with *this lesson*.

- **Exercise 1:** Sketch and describe solids of revolution.
- **Exercise 4:** Find surface areas and volumes of solids of revolution.
- **Exercise 6:** Form solids of revolution in the coordinate plane.

Re-engage (TIER 2)

Identify where students need support with *prior skills*.

- Find volumes and surface areas of cylinders, cones, and spheres. **Geometry Sections 12.2, 12.4, and 12.5**

Intervene (TIER 3)

Identify where students have additional *unfinished learning*.

- Describe and draw cross sections of solids. **Geometry Section 12.1**

Deepening Student Learning

Support proficient and advanced learners with suggested resources targeted to promote more complex thinking and deeper understanding of lesson concepts.

Differentiated Resources

Learning is individualized and students may move in and out of proficiency levels with each skill and concept. The online resources provide a wide range of layered support for the diverse needs in your classroom.

RESOURCES

India's Notes
Equity in Action

Some students may not connect with you because you are of a different culture or ethnic group than they are. Find ways to connect with these students by asking questions and actively listening when they talk. Try to find something you both have in common. Take time each day to welcome all students to your classroom and model fair rules and procedures so all students feel connected.

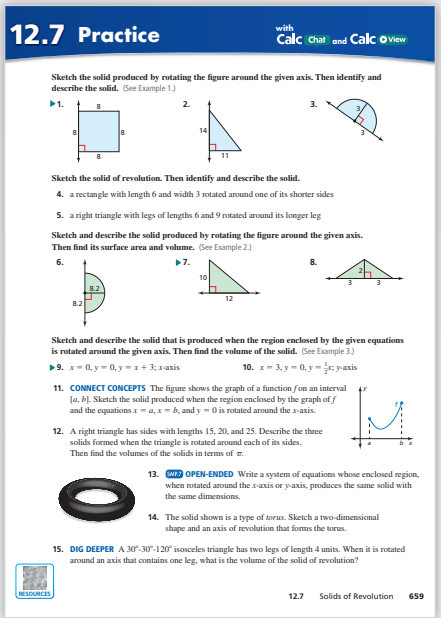
Listen to India talk about her insights and inspiration on equity in action.

12.7 Practice

See the *Answer Guide* for all Practice notes and answers to help inform next steps for your students.

Data Talk

See the *Answer Guide* for suggestions on how to connect the data display to math and facilitate classroom discussion.


12.7 Practice

with Calc Chat and Calc View

Sketch the solid produced by rotating the figure around the given axis. Then identify and describe the solid. (See Example 1.)

1. 8, 8, 8
2. 14, 11
3. 3, 3

Sketch the solid of revolution. Then identify and describe the solid.

4. a rectangle with length 6 and width 3 rotated around one of its shorter sides
5. a right triangle with legs of lengths 6 and 9 rotated around its longer leg

Sketch and describe the solid produced by rotating the figure around the given axis. Then find its surface area and volume. (See Example 2.)

6. 8.2, 8.2
7. 10, 12
8. 2, 3, 3

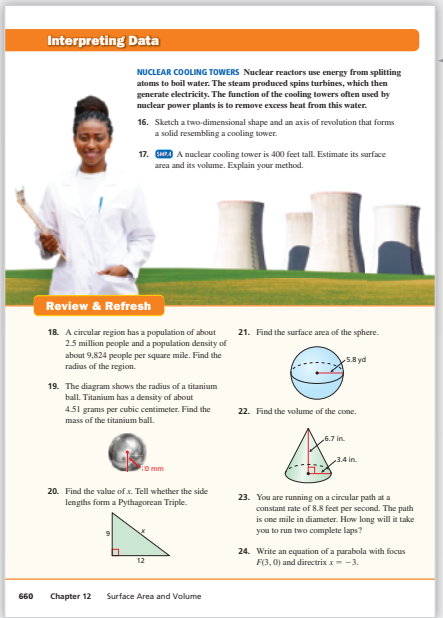
Sketch and describe the solid that is produced when the region enclosed by the given equations is rotated around the given axis. Then find the volume of the solid. (See Example 3.)

9. $x = 0, y = 0, y = x + 3$; x-axis
10. $x = 3, y = 0, y = \frac{1}{2}x$; y-axis

11. **CONNECT CONCEPTS** The figure shows the graph of a function f on an interval $[a, b]$. Sketch the solid produced when the region enclosed by the graph of f and the equations $x = a$, $x = b$, and $y = 0$ is rotated around the x-axis.
12. A right triangle has sides with lengths 15, 20, and 25. Describe the three solids formed when the triangle is rotated around each of its sides. Then find the volumes of the solids in terms of π.
13. **MP7 OPEN-ENDED** Write a system of equations whose enclosed region, when rotated around the x-axis or y-axis, produces the same solid with the same dimensions.
14. The solid shown is a type of *torus*. Sketch a two-dimensional shape and an axis of revolution that forms the torus.
15. **DIG DEEPER** A 30°-30°-120° isosceles triangle has two legs of length 4 units. When it is rotated around an axis that contains one leg, what is the volume of the solid of revolution?

RESOURCES

12.7 Solids of Revolution 659


Interpreting Data

NUCLEAR COOLING TOWERS Nuclear reactors use energy from splitting atoms to boil water. The steam produced spins turbines, which then generate electricity. The function of the cooling towers often used by nuclear power plants is to remove excess heat from this water.

16. Sketch a two-dimensional shape and an axis of revolution that forms a solid resembling a cooling tower.
17. **MP7** A nuclear cooling tower is 400 feet tall. Estimate its surface area and its volume. Explain your method.

Review & Refresh

18. A circular region has a population of about 2.5 million people and a population density of about 9,824 people per square mile. Find the radius of the region.
19. The diagram shows the radius of a titanium ball. Titanium has a density of about 4.51 grams per cubic centimeter. Find the mass of the titanium ball.

10 mm

20. Find the value of x. Tell whether the side lengths form a Pythagorean Triple.

9, x, 12

21. Find the surface area of the sphere.

5.8 yd

22. Find the volume of the cone.

6.7 in., 3.4 in.

23. You are running on a circular path at a constant rate of 8.8 feet per second. The path is one mile in diameter. How long will it take you to run two complete laps?
24. Write an equation of a parabola with focus $F(3, 0)$ and directrix $x = -3$.

660 Chapter 12 Surface Area and Volume

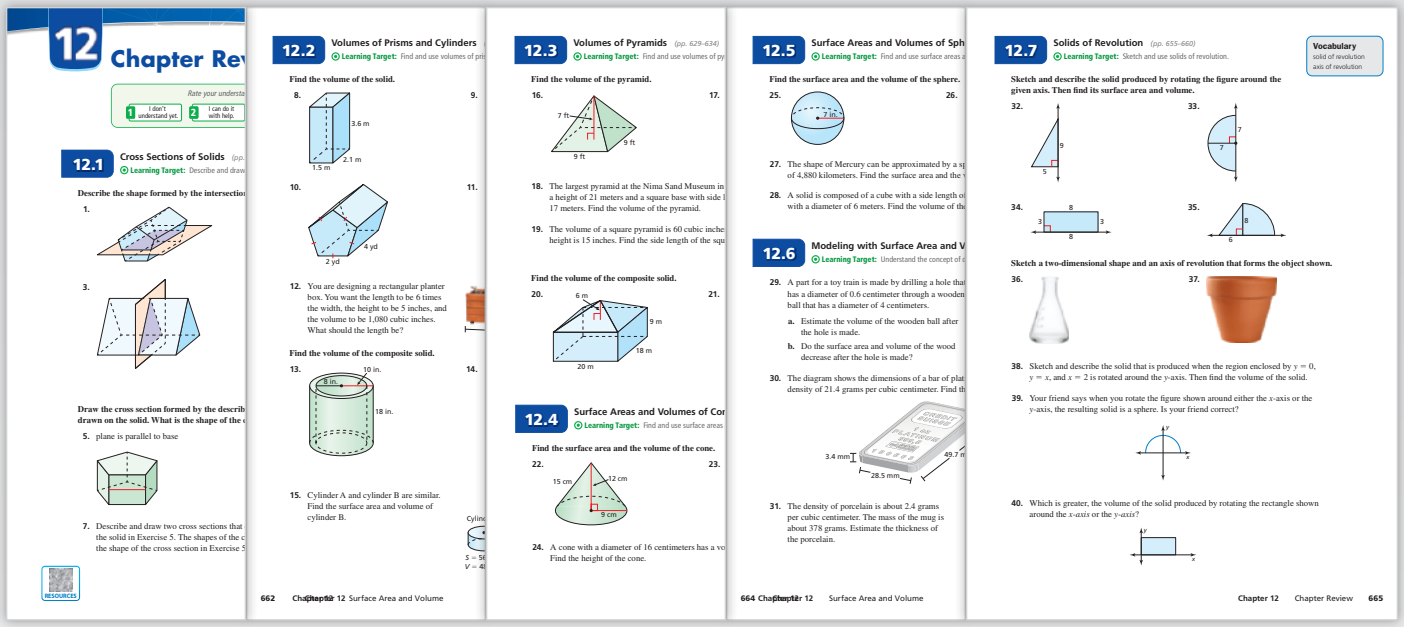
Student Edition pages 659 and 660

Chapter Review

CHAPTER 12

Supporting Students

See the *Answer Guide* for all Chapter Review notes and answers. Encourage students to use the **Self-Assessment** to rate where they are in understanding each Learning Target.



Student Edition pages 661–665

Differentiated Resources

Access suggested resources to reinforce or reteach concepts from this chapter. The resources offered online provide a wide range of layered support for the diverse needs in your classroom.

RESOURCES

My Thoughts on the Chapter

Go online to reflect on this chapter.
How did you support student learning?

What worked? What didn't work?

What would you do differently?

CHAPTER 12 Performance Task

Paul's Notes

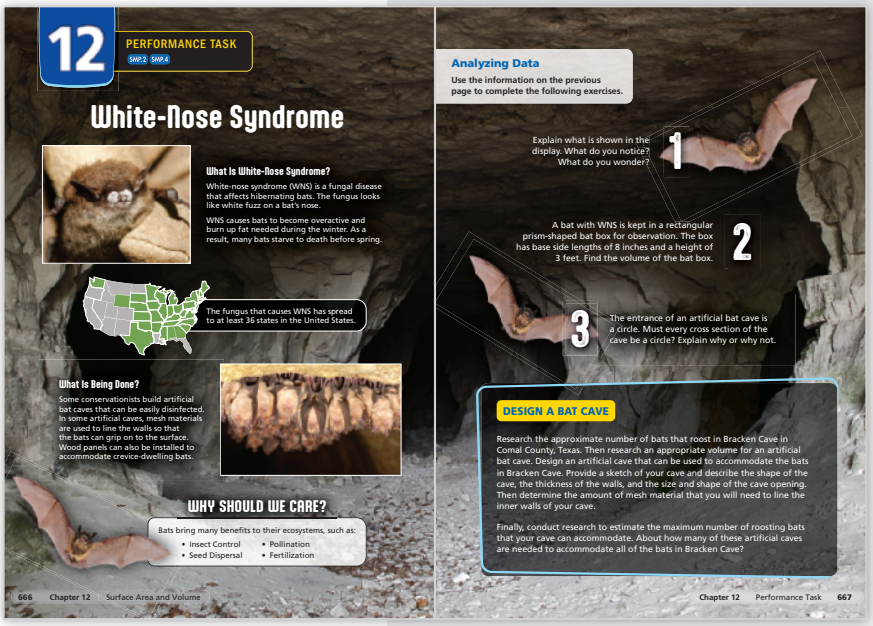
Conclude the Chapter with the Career

This Performance Task asks students to apply their understanding of surface area, volume, and population density to design an artificial cave to accommodate bats.

White-Nose Syndrome

? "Have you ever seen a bat? If so, where? What do you know about bats?"

- Ask students if they are aware that bat populations are declining due to white-nose syndrome. Many students may wonder why this is relevant to them. Direct students to the text at the bottom of Student Edition page 666, and invite them to research other benefits that bats bring to their ecosystems.
- Discuss students' responses to Exercise 1 as a class. Then have them complete Exercises 2 and 3 independently.
- Have students independently gather information about Bracken Cave and appropriate volumes of artificial bat caves before working in small groups to complete the rest of the task.



Student Edition pages 666 and 667

Modeling Process

SMP.4 Students will use a model to represent their proposed artificial bat cave. Using their model, students will estimate how much mesh material is needed to line the walls of the cave, as well as how many bats the cave can accommodate.

? **Formulate:** Students can be creative when designing an artificial cave. Some students may consider a composite solid rather than a simple rectangular prism. "What are the important features of your artificial cave? Which ones will impact the number of bats the cave can accommodate?"

? **Compute:** Students will determine the dimensions for their cave and the amount of mesh material needed to line the walls. "Based on your research, does your cave have an appropriate volume? How do you know?"

? **Interpret:** Ask students to consider why the volume and surface area of the cave are important. "Does the size of the cave opening impact the amount of mesh material that is needed? Can you alter your design to decrease the surface area without changing the volume? Why might that be a good strategy?"

- **Validate:** In small groups, encourage students to share how they estimated the maximum number of bats that their artificial cave can accommodate. Each student should share their design, explain the volume and amount of mesh material needed, and report how many of their artificial caves are needed to accommodate all of the roosting bats in Bracken Cave. Ask students to compare answers and validate the work of their group members.

EL English Learner Support

Constructing Mathematical Explanations

- Have EL students translate this task if needed so they are able to understand the context.
- They may also write their answers in their home language and then use a device to translate them into English. Support students with grammatical errors as needed.
- Have students work with native speakers in small groups or pairs to complete the exercises and Design a Bat Cave.
- If EL students will be presenting their work, allow them time to practice reading their answers aloud.